Article

Magnitlangan kvazi – Schwarzschild qora o’ralari atrofida zaryadlangan zarralar va magnit dipollarning dinamikasi

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Xulosa: Umumiy nisbiylik nazariyasi klassik nazariya bo‘lgani uchun muammolardan istisno emas.Asosiy muammolardan biri bu singularlik.Qora o‘ralar markazidagi singularlik yoki kosmologik singularlik.Bu singularliklar cheksiz qiymatlar beradi.Masalan kosmologiyada boshlang‘ich nuqtasida cheksiz egrilik, cheksiz zichlik, cheksiz temperatura bo‘ladi.Demak kosmologik singularlik umumiy nisbiylik nazariyasida bor.Astrofizik nuqtai nazardan qora o‘ralar markazida singularlik bo‘ladi.Bu yerda Kretschmann skalyarini xisoblasak egrilik markazda cheksizlikka ketadi.Lekin cheksizlik fizikada muammolar bor ekani bildiradi va umumiy nisbiylik nazariyasini xamma holda xam qo‘llab bo‘lmasligini ko‘rishimiz mumkin. Qora o’ralarni matematik jihatdan tavsiflashda ham umumiy nisbiylik nazariyasi doirasidagi va alternativ gravitatsiya nazariyalaridagi turli yechimlar mavjud bo’lib bunday yechimlarni nazariy jihatdan sinov zarrachalarning qora o’ra atrofidagi harakatida o’zini qanday tutishiga qarab o’rganish mumkin. Qora o’ra atrofida magnit maydon mavjud deb qolaversa zarrachalar elektr zaryadiga yoki magnit xususiyatlarga ega deb qarash mumkin. Shu jihatdan bunday jarayonlarni o’rganish gravitatsiya nazariyalarini tekshirishda xususan qora o’ralar tabiatini o’rganishda muhim ahamiyat kasb etadi. Tashqi magnit maydonda joylashgan aksial-simmetrik qora o’ralar atrofidagi elektromagnit maydonlarni hamda zaryadli va sinov zarralar harakatida zarralar effiktiv potensiallarining qanday o’zgarishini o’rganish va shu bilan bir qatorda Kerr-Taub-NUT metrikasi misolida ko’rib chiqishdan iborat

Maqsad. Qora o’ra atrofidagi zarralar harakatini o’rganish orqali kvazi-Schwarzschild va konform gravitatsiya sharoitidagi qora o’ra yechimlarini o’rganish.

Materiallar va usullar. Umumiy nisbiylik nazariyasida makroskopik elektrodinamikaning matematik apparati; zarra va maydonning harakat tenglamalarini yechishning analitik va raqamli usullari. Yechimlarda qatnashadigan fazo-vaqt parametrlarining zarralar harakatiga ta’sirini o’rganish; bunday yechimlar umumiy nisbiylik nazariyasidagi yechimlardan qanchalik farq qilishi yoki ularning effektla - rini namoyon qila olish qobiliyatini o’rganish; tashqi magnit maydonning mavjudligi masalaga qanchalik hissa qo’shishini baholash

Natijalar. Kvazi-Scwarzschild va conformgravitatsiya sharoitidagi qora o’ralar tashqi magnit maydoniga kiritilgan hollar uchun zarralar harakati birinchi marta o’rganildi va bu yechimlar qaralgan effektlarning umumiy nisbiylik nazariyasidagi yechimlardan qay darajada farqlanishi ko’rib chiqildi. Bunda tashqi elektromagnit maydonning bu effektlarga ta’siri qaralgan yechimlar uchun birinchi marta baholandi. Kvazi-Schwarzschild yechimning deformatsiya parametri tashqi magnit maydon bo’lgan va bo’lmagan hol uchun umumiy nisbiylik nazariyasidagi Kerr yechimining spin parametrini qanchalik o’rnini bosishi baholandi.

Kalit so’zlar: qora o’ra, kvazi-Schwarzschild, konform gravitatsiya, spin, Kerr-Taub, elektromagnit, effektiv potensial.

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Dynamics of charged particles and magnetic dipoles around magnetized quasi-Schwarzschild black holes

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Abstract: Since the theory of general relativity is a classical theory, it is not without its challenges. One of the main issues is singularity, such as the singularity at the center of black holes or cosmological singularity. These singularities yield infinite values. For example, in cosmology, the initial point has infinite curvature, infinite density, and infinite temperature. This indicates that cosmological singularities exist within the framework of general relativity. From an astrophysical perspective, there is a singularity at the center of black holes. When calculating the Kretschmann scalar, curvature at the center tends to infinity. However, infinities in physics suggest the presence of fundamental problems, demonstrating that general relativity cannot be applied universally in all cases. In the mathematical description of black holes, there are various solutions both within the framework of general relativity and in alternative theories of gravity. These solutions can be theoretically studied by analyzing the behavior of test particles around black holes. Assuming the presence of a magnetic field around a black hole and considering that particles may possess electric charge or magnetic properties, the study of such processes becomes important for testing theories of gravity and understanding the nature of black holes.This research involves examining the electromagnetic fields around axially symmetric black holes situated in an external magnetic field and analyzing how the effective potentials of charged and test particles change during their motion. Additionally, this study considers the example of the Kerr-Taub-NUT metric.
Background. Studying the motion of particles around a black hole allows for the exploration of black hole solutions in quasi-Schwarzschild and conformal gravity conditions.
Materials and methods. The mathematical apparatus of macroscopic electrodynamics in general relativity involves analyzing the motion equations of particles and fields, using both analytical and numerical methods for solutions. This includes studying the impact of spacetime parameters involved in the solutions on particle motion and evaluating how these solutions differ from those in general relativity or their ability to demonstrate observable effects. Additionally, assessing the contribution of an external magnetic field to the problem is crucial to understand its influence on the dynamics and behavior of particles and fields within the framework of general relativity.
Results. The motion of particles around quasi-Schwarzschild and conformal gravity black holes in the presence of an external magnetic field was studied for the first time, and the extent to which these solutions differ from those in general relativity was analyzed. The influence of the external electromagnetic field on these effects was evaluated for the first time for the considered solutions. The deformation parameter of the quasi-Schwarzschild solution was assessed in terms of how well it can substitute for the spin parameter of the Kerr solution in general relativity, both with and without the presence of an external magnetic field.

Keywords: black hole, quasi-Schwarzschild, conformal gravity, spin, Kerr-Taub, electromagnetic, effective potential.

Kirish

Hozirgi vaqtda neytron yulduzlarini aniqlashga urinishlarga qaramasdan somon yoʻli markazida gravitatsion hamkorlik orqali ultra massivli qora tuynuk Sagittarius A* yoy yaqinidagi qayta ishlangan radio modullari sifatida, ularning astrofizik kuzatuvlar yoʻq. SgrA* atrofida pulsarlarning yoʻqliging sabablaridan biri SMQT atrofidagi plazma muhitida radio toʻlqinlarning tarqalishi, ikkinchisi esa neytron yulduzining dipol momenti va markaziy qora oʻra magnit zaryadlar yoki elektr toki natijasida hosil boʻlgan qora oʻra atrofidagi magnit maydon oʻrtasidagi magnit maydonning ta'siri boʻlishi mumkin. Neytral zarrachalarning barqaror aylanish va xaotik harakati, statik va aylanadigan qora oʻralar atrofida zaryadlangan zarralalarning dinamikasi va kvazigarmonik tebranishlari tashqi asinxron bir xil magnit maydonlarga va plazma magnitosferasii turli xil qora oʻralarni oʻrab oladi. Xususan, Lyapunov usuli yordamida muntazam va xaotik orbitalar oʻrtasidagi farqni koʻrsatish mumkin. Bundan tashqari, kichik bir tizimsiz va tortishish effektlari xaotik harakatining pasayishiga olib keladi.

Ushbu ishda bizning asosiy maqsadimiz magnitlangan kvazi-Schwarzschild qora oʻra atrofida zaryadlangan zarralar va magnit dipollarning harakatini oʻrganishdir. Ish quyidagicha tashkil etilgan: qora oʻraning atrofida zaryadlangan zarrachalarning dinamikasini va Kerr fazo vaqtiga nisbatan oʻrganishga bagʻishlangan. Magnit dipolining tashqi magnit maydonga tushirilgan qora oʻraning kvazi-Schwarzschild atrofida harakatlanishi oʻrganiladi. Ushbu boʻlimda olingan natijalar Kerr qora oʻra atrofidagi zarrachalarning dinamikasi bilan taqqoslanadi. Ish davomida biz $G = 1 = c$ boʻlgan boʻshliqqa oʻxshash belgi $(-, +, +, +)$ va birlik tizimdan foydalanamiz.

Natijalar

Kompakt obyekt atrofidagi magnit maydon magnitlangan zarralarning tashqi asimptotik tarzda bir xil magnit maydonga tushirilgan Kvazishvartschild kompakt obyekti atrofida harakatlanishini oʻrganishdan oldin, biz zarraning faqat elektr bilan zaryadlanganligi bilan boshlaymiz. Schwarzschild fazo vaqtning yarim metrikasi Kerning aylanadigan kvazimetrikasidan [4] quyidagi ifoda yordamida olinishi mumkin:

$$g_{\mu\nu} = g_{\mu\nu}^{\text{Schw}} + \epsilon h_{\mu\nu}$$

Bu yerda $g_{\mu\nu}^{\text{Schw}}$ Schwarzschild standart metrikasiga va $\epsilon h_{\mu\nu}$ Schwarzschild makonidan chetga chiqishga mos keladi. Parametr sferik nosimmetrik boʻshliqdan oʻgʻishni belgilaydi. Tortishish obyektining Q kvadrat momentidagi qoʻshimcha qismlarni tufayli vaqt

$$Q = -\epsilon M^3$$

U manfiy va musbat qiymatlarni qabul qilishi mumkin [4]. Qarama-qarshi komponentlarda chiziqli yondashuvda fazo-vaqt metrikasi quyidagi yozilishi mumkin

$$g_{\mu\nu} = g_{\mu\nu}^{\text{Schw}} - \epsilon h_{\mu\nu}$$

Va shunday qilib, $h^{\mu\nu}$ ning yuqori koʻrsatkichlari Schwarzschild metrik tenzor bilan kamaytirilishi mumkin. $h^{\mu\nu}$ ning qarama-qarshi komponentlari ifodalari bilan belgilanadi [4].

$$h^{tt} = f^{-1}(1 - 3\cos^2\theta)F_1$$

$$h^{tt} = f(1 - 3\cos^2\theta)F_1$$

$$h^{\theta\theta} = -\frac{1}{r^2}(1 - 3\cos^2\theta)F_2$$

$$h^{\phi\phi} = -\frac{1}{r^2\sin^2\theta}(1 - 3\cos^2\theta)F_2$$

$$f = 1 - \frac{2M}{r}$$

Bu yerda F_1 va F_2 radial funksiyalar:

$$F_1 = -\frac{5(r - M)}{8Mr(r - 2M)}(2M^2 + 6Mr - 3r^2) - \frac{15r(r - 2M)}{16M^2}\ln\frac{r}{r - 2M} \tag{1.1}$$

$$F_2 = \frac{5(2M^2 - 3Mr - 3r^2)}{8Mr} + \frac{15(r^2 - 2M^2)}{16M^2}\ln\frac{r}{r - 2M} \tag{1.2}$$

$h^{\mu\nu}$ indekslarini ishlatishdan keyin yarim Schwarzschild fazo-vaqtning $g_{\mu\nu}^{\text{Schw}}$ metrikasi quyidagi koʻrinishga ega boʻladi.

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + k(r,\theta)r^2 d\Omega^2 \tag{1.3}$$

Bu yerda

$$g_{tt} = -f\left[1 - \epsilon F_1\left(1 - 3\cos^2\theta\right)\right] \tag{1.4}$$

$$g_{rr} = f^{-1}\left[1 + \epsilon F_1\left(1 - 3\cos^2\theta\right)\right] \tag{1.5}$$

$$k(r,\theta) = 1 - \epsilon F_2\left(1 - 3\cos^2\theta\right) \tag{1.6}$$

Fazo vaqt metrikasining ϵ kvazi-Schwarzschild ta'siri uchun kerak bo'lgan qismini ta'minlash uchun mutanosibdir. $\epsilon = 0$ holatida Schwarzschild vaqti qayta tiklanganligini osongina tekshirish mumkin. Shuni ta'kidlash kerakki, $g^{rr} = 0$ holati $r_h = 2m$ da voqealar gorizontining joylashuvini beradi, Schwarzschild qora tuynugi bilan bir xil.

Vald [12] usuli yordamida elektromagnit maydonlarning to'rt vektorli potensialning tarkibiy qismlarini topish mumkin.

$$A_\mu = \left(0, 0, 0, \frac{1}{2}B\right) \tag{1.7}$$

Metrikni (1.3) dan foydalanib, kovariant qismlarini quyidagi shaklda yoziladi.

$$A_\mu = \left\{0, 0, 0, \frac{1}{2}Bk(r,\theta)r^2\sin^2\theta\right\} \tag{1.8}$$

Kvazi-Schwarzs kompakt obyekti atrofida magnit maydon ifodani topish mumkin va haqiqiy kuzatuvlar to'rtinchi tezlikni topishga yordam beradi

$$U_\alpha = \left\{\left(f\left[1 + \frac{1}{2}\epsilon F_1\left(1 + 3\cos^2\theta\right)\right]^{-\frac{1}{2}}, 0, 0, 0\right)\right\} \tag{1.9}$$

Keyin, magnit maydonning tanlangan ramkaga nisbatan ortonormal tarkibiy qismlari quyidagi ko'rinishni oladi [21]:

$$\hat{B}^r = B\cos\theta\frac{1 + \epsilon F_2(3\cos 2\theta - 1)}{1 + \frac{\epsilon}{2}F_2(3\cos 2\theta + 1)} \tag{1.10}$$

$$\hat{B}^\theta = B\sin\theta\sqrt{f}\sqrt{\frac{1 + \frac{\epsilon}{2}F_1(3\cos 2\theta + 1)}{v1 + \frac{\epsilon}{2}F_2(3\cos 2\theta + 1)}} \times \sqrt{\frac{1 + \frac{\epsilon}{4}\left[rF_2' + 2F_2\right](3\cos 2\theta + 1)}{1 - \frac{\epsilon^2}{4}F_1^2(3\cos 2\theta + 1)^2}} \tag{1.11}$$

$$\hat{B}^\phi = 0 \tag{1.12}$$

Asosiy raqam radial koordinatalar bo'yicha xususiy degan ma'noni anglatadi. Sof Schwarzschild fazo vaqtida quyidagi shaklini olishni ko'rish osondir.

$$\hat{B}^r = B\cos\theta \tag{1.13}$$

$$\hat{B}^\theta = B\sqrt{f}\sin\theta \tag{1.14}$$

Nyutonning kuchsiz maydon tuzumida $\frac{M}{r} \to 0$ magnit maydonning tarkibiy qismlari bo'ladi.

$$\hat{B}^r = B\cos\theta \quad \hat{B}^\theta = B\sin\theta \tag{1.15}$$

Nyuton chegarasi bilan mos keladi.

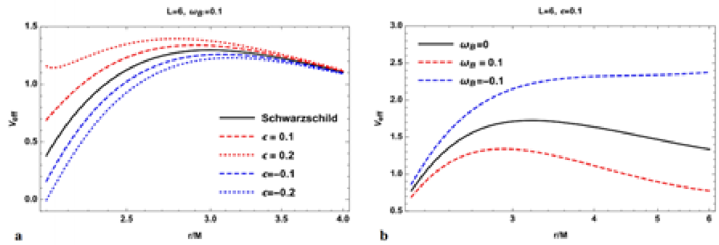

Figure 1. For M=1, the radial coordinate is the effective potential as a function of R. The left panel corresponds to a change in the deformation paramatry, the right panel is designed to change the magnetic effect.
Rasm 1. M=1 uchun radial koordinata r funksiyasi sifatidagi effektiv potensial. Chap panel deformatsiya paramatrining o'zgarishiga mos keladi, o'ng panel magnit ta'sirini o'zgartirish uchum mo'ljallangan.

Kvazi Schwarzschild kompakt obyektining atrofida zaryadlangan zarrachaning harakat tenglamasni qisqacha o'rganamiz. Bu holatda bo'lgani kabi, markaziy obyektlar atrofida aylanadigan zarralar uchun Hamilton-Jacobian harkatining tenglamasini ishlatish qulayroq. Tashqi elektromagnit maydon mavjud bo'lganda, tenglama quyidagicha yoziladi

$$g^{\alpha\beta}\left(\frac{\partial S}{\partial x^{\alpha}}+eA_{\alpha}\right)\left(\frac{\partial S}{\partial x^{\beta}}+eA_{\beta}\right)=-m^2 \tag{1.16}$$

bu yerda e va m mos ravishda elektr zaryadi va o'rganilayotgan zarrachaning massasi. (1.16) harakat tengligi sistema integral bo'lmaganda ajratilmaydi. Bu holda (1.16) tenglama Gamilton formalizmi bilan almashtirilishi kerak. Biroq, sinov zarrachasining ekvatorial tekislikdagi harakati tekshirilganda (1.16) tenglama quyidagicha ajratilgan shakldа ifodalanishi mumkin:

$$S=-Et+L\phi+S_r+S_\theta \tag{1.17}$$

bu yerda E va L o'rganilayotgan zarrachaning energiya va impuls momenti birlik massasiga bog'liqligini aniqlaydi. Shunday qilib, sinov zarrachasining birlik massasi bilan harakatlanish tenglamasi quyidagicha yoziladi:

$$\left(\mathcal{L}+\frac{eB}{2m}r^2(1-\epsilon F_2)\right)^2\frac{1}{r^2(1-\epsilon F_2)}+\frac{f}{1+\epsilon F_1}\left(\frac{\partial S}{\partial r}\right)^2-\frac{\epsilon^2}{f(1-\epsilon F_1)}=-1 \tag{1.18}$$

bu yerda $\mathcal{E}=E/m$ va $\mathcal{L}=L/m$ singular energiya va sinov zarrachasining impuls momenti. Ekvatorial tekislikda harakatlanayotgan sinov zarralari uchun $\theta=\pi/2$ radial harakatda effektiv potensial quyidagicha bo'lishi mumkin.

$$\dot{r}^2=\mathcal{E}^2-V_{\text{eff}} \tag{1.19}$$

$$V_{\text{eff}}=f(1-\epsilon F_1)\left\{1+\left(\frac{L}{k(r,\frac{\pi}{2})r^2}-\omega_B\right)^2\right\} \tag{1.20}$$

Zaryadlangan zarrachaning siklotron chastotasini aniqlaydigan $\omega_B=\frac{eB}{2mc}$ elektr zaryadlangan zarracha va tashqi magnit maydon o'rtasidagi o'zaro ta'sirga mos keladi. Shubhasiz o'g'ish parametrining o'sishi ham effektiv potensialni oshiradi, magnit maydon qarama-qarshi ta'sirga ega bo'lsa-da. Ekvatorial tekislikdagi zarrachaning aylana harakati uchun quyidagi standart shartlar belgilanishi mumkin:

$$V_{\text{eff}}(r)=E^2 \quad \text{va} \quad V'_{\text{eff}}(r)=0 \tag{1.21}$$

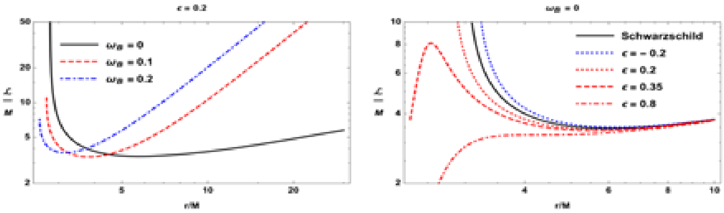

Figure 2. Different values of magnetic interaction (left panel) and deformation for the parameter (right panel), the dependence of the momentum of the particle under study on the radius.

Rasm 2. Magnit o'zaro ta'sirning turli qiymatlari (chap panel) va deformatsiya parametri (o'ng panel) uchun o'rganilayotgan zarrachaning impuls momentining radiusga bog'liqligi.

Bu zaryadlangan sinov zarrachasining impuls momenti 2-rasmda ko'rsatilgandek radial bog'liqlikka ega bo'lishiga olib keladi. Biz o'ng panelda $\epsilon = 0$ bo'lgan chiziq uchun Schwarzschildning odatiy shaklini ko'ramiz. Biroq, $\epsilon \approx 0.32$ atrofida o'g'ish parametrining ma'lum qiymatdan boshlab, u chiziqlar shaklini o'zgartiradi, bu esa ular uchun mos keladigan radiusda maksimal nuqtaga ega bo'lishiga olib keladi, bu esa $\epsilon \approx 0$ boshliqqa nolga aylanadi. Biz bu masalani keyinchalik ISCO radiusini aniqlashning muhim tavsifini o'z ichiga olgan keying kichik bo'limga qaytaramiz.

Ushbu kichik qismda asimptotik bir xil magnit maydonga tushurilgan Kvazishvartschild kompakt obyekti atrofida ichki barqaror aylana orbitalar (ISCO) deb nomlangan kattaliklarni o'rganamiz. Hisoblashlar asosida biz ϵ va B parametrlarning Kerr yechimining aylanish parametri qanday darajada o'rnini bosishi mumkinligi haqidagi savolga javob berishga harakat qilamiz. Mulohaaza agar yuqorida ko'rsatilgan parametrlari Kerring qora tuynugikning aylanish parametrini o'rnini to'ldirsa, u holda bir xil ISCO radiusi uchun aylanish parametri va ϵ va B parametrlari o'rtasidagi munosabatni o'rganamiz. Buning uchun (1.21) shartga qo'shimcha kattaliklar qo'shishingiz mumkin.

$$V''_{\text{eff}}(r) = 0 \tag{1.22}$$

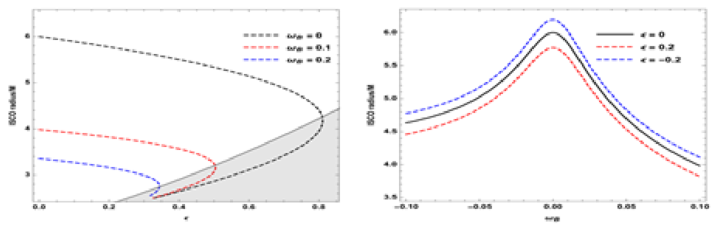

Figure 3. Different values of magnetic interaction (left panel) and deformation for the parameter (right panel), the dependence of the momentum of the particle under study on the radius.

Rasm 3. Magnit o'zaro ta'sirning turli qiymatlari (chap panel) va deformatsiya parametri (o'ng panel) uchun o'rganilayotgan zarrachaning impuls momentining radiusga bog'liqligi.

3-rasmda ko'rsatilgandek ushbu uchta shartni hisobga olgan holda (1.21) bilan birga ISCO radiusi ϵ va B parametrlari o'rtasidagi o'zaro bog'liqlik jadvalini tuzish mumkin. Harflardan ϵ va B ning ikkita parametrining o'sishi zaryadlangan sinov zarrachasining ISCO radiusini kamaytiradi. Chap panelda aniq ko'rinib turibdiki, agar siz ϵ parametrini muayyan qiymatlarda oshirsangiz, ISCO radiusi pastga tushishdan oldin boshlang'ich qiymatlarini qabul qilish o'rniga kichikroq va kichikroq bo'ladi. Vaziyatni aniqlashtirish uchun sinov zarrachasining impuls momentining holatini hisobga olish kerak, bu zarrachaning oxirgi barqaror aylana orbitidagi harakatlanishi uchun impuls momentining bu orbitadagi radiusda minimal bo'lishi kerakligini bildiradi. Ushbu shartdan foydalanib, sinov zarrachasining impuls momentini aylana orbitaning radiusiga qarab, o'ng paneldagi 2-rasmda ko'rsatilgandek jadvalga murojaat qilishingiz mumkin.

Biz burilish parametri 0.32 ˘ 0.8 oralig'ida bo'lsa, tashqi magnit maydon yo'qligida impuls momenti maksimal qiymatlarga ega bo'ladi (oldingi bo'limda aytgan edik). Lekin 3-rasmni chizishda faqat sinov zarrachasining impuls momenti ekstremumga ega bo'lgan ishi o'lladik. Shunday qilib, 3-rasmning chap panelidagi biz ushbu ekstremum nuqtalaridan bunday o'zishni chiqarib, tashlashimiz kerak, bu esa ISCO chiziqlarining pastki qismini maydon tomonidan ko'rsatilgandan burilish nuqtalaridan boshlash kerak. Magnit maydon ishi kuchsiz bo'lsa, kutilgandek oldga borib bo'lib, va'ni magnit momentining oshishi bilan Lorentz kuchi kuchayadi, bunda ISCO radiusi kamayadi. Va nihoyat, biz ushbu kichik bo'limning boshida berilgan savolga javob berishni rejalashtirmoqdamiz: ϵ va B parametrlari Kerr metrikasining aylanish parametrini qanday o'rnini bosishi mumkinligi 4-rasmda ko'rsatilgan.

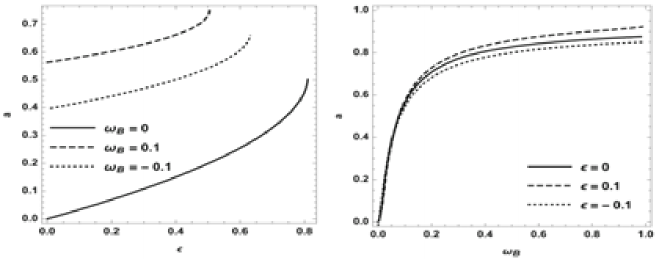

Figure 4. Left panel: the deformation parameter for different values of the magnetic parameter can replace the spin parameter. Right panel: how much the magnetic parameter can replace the spin parameter for different values of the deformation parameter.

Rasm 4. Chap panel: magnit parametrining turli qiymatlari uchun deformatsiya parametri spin parametrini o'rnini bosa olishi. O'ng panel: deformatsiya parametrining turli qiymatlari uchun magnit parametri spin parametrini qanchalik o'rnini bosa olishi.

ISCO aylanish parametrining ta'siri ostida Kerr metrikasida qanday harakat qilishini bilish uchun ushbu parametrlar orasidagi degeneratsiya grafigkasini chizishingiz mumkin. ISCO ning ϵ parametrega bog'liqligi tufayli, bu parameter aylanish parametrini to'liq o'rnini bosa olmasligi mumkin, chunki ISCO radiusi M ga intilmagan, bu esa Kerr metrikasida $a \to 1$ ning aylanishi uchun sodir bo'ladi. Endi bu parametr tashqi magnit maydon mavjud bo'lmaganda, aylanish parametrini faqat ≈ 0.5 ga o'rnini bosishi mumkinligini ko'rish mumkin. O'ng panelda shuningdek, magnit maydon va burilish parametri mavjud bo'lmaganda, mimik diapazoni $a > 0.9$ dan oshib, tez aylangan Kerr fazo-vaqti bilan taqqoslanadi.

Magnit dipol harakati: Kerr va kvazi-Schwarzschild qora o'ralarini solishtirish

Biz magnit dipolining harakatini asimptotik bir xil magnit maydonga kiritilgan Kvazishvartschild kompakt obyekti atrofida ko'rib chiqamiz. Hamilton-Jacobi magnit dipol harakati tenglamasi quyidagi shaklni oladi:

$$g^{\mu\nu}\frac{\partial S}{\partial x^{\mu}}\frac{\partial S}{\partial x^{\nu}} = -\left(m - \frac{1}{2}D^{\mu\nu}F_{\mu\nu}\right)^2 \tag{1.23}$$

Bu yerda $D^{\mu\nu}$ zarraning elektrodinamik xususiyatlarini aniqlaydigan asimmetrik polarizatsiya tenzordir. Biz zarrachani elektr neytral $q = 0$ hisoblaymiz deb hisoblaymiz va polarizatsiya tenzori faqat μ magnit moment tomonidan tasvirlangan.

Bu yerda ta'kidlash lozim, magnit dipolining lektr zaryad bilan harakatini muqobil ravishda o'rganish mumkin. (1.16) tenglama o'rniga (1.23) tenglamaning chap qismini olish kerak. Biroq, bu ishda biz magnit dipollarining harakatini magnitlanmagan neytron yulduzlar supermassive qora tuynuklar atrofida aylantirishni mo'gasi qilmagandamiz va neytron yulduzi o'zgarmas magnit dipol momently elektr neytral sinov zarrasi deb hisoblash mumkin. Odatda supermassive qora tuynuk massasi neytron yulduzining massasidan ancha katta bo'lgani uchun bu neytron yulduzini birinchi marta fazoda harakatlanadigan sinov zarrasi sifatida ishlatish imkon beradi. Shuning uchun biz faqat elektr neytral magnit dipolining harakatiga e'tibor qaratamiz. Bu holda, bu tenzor komponentlari shaklida yozilgan bo'lishi mumkin [15].

$$D^{\mu\nu} = \eta^{\mu\nu\alpha\beta}u_{\alpha}u_{\beta} \tag{1.24}$$

Quyidagi shartni qanoatlantiradigan:

$$D^{\mu\nu}u_\nu = 0 \tag{1.25}$$

Bu yerda magnit dipolining magnit to‘rt-impulsini tasvirlaydi. μ_α — elektromagnit maydon tenzor, bundan tashqari, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ elektromagnit maydon tarkibiy qismlari sifatida qayd qilinishi mumkin.

$$F_{\mu\nu} = -\eta_{\mu\nu\alpha\beta}B^\alpha u^\beta + 2u_{[\mu}E_{\nu]} \tag{1.26}$$

Bu yerda $\eta_{\alpha\beta\gamma\nu}$ ni $\epsilon_{\alpha\beta\gamma\nu}$ Levi-Civitanning psevdotenzoz shakli deb belgilang:

$$\eta_{\alpha\beta\gamma\nu} = \sqrt{-g}\epsilon_{\alpha\beta\gamma\nu} \quad \text{va} \quad \eta^{\alpha\beta\gamma\nu} = -\frac{1}{\sqrt{-g}}\epsilon^{\alpha\beta\gamma\nu} \tag{1.27}$$

$$g = \det|g_{\mu\nu}| = -r^4\sin^2\theta \quad \text{bilan fazovaqt metrikasi uchun (1.3)}$$

$$\begin{aligned} \epsilon_{\alpha\beta\gamma\nu} &= +1, \quad \text{bir xil almashtirishlar uchun} \\ \epsilon_{\alpha\beta\gamma\nu} &= -1, \quad \text{har xil almashtirishlar uchun} \\ \epsilon_{\alpha\beta\gamma\nu} &= 0, \quad \text{boshqa xil almashtirishlar uchun} \end{aligned} \tag{1.8}$$

(1.24) va (1.25) ni (1.26) dan foydalanib yozish mumkin:

$$D^{\mu\nu}F_{\mu\nu} = 2\mu^\alpha B_\alpha = 2\mu^{\hat{\alpha}}\hat{B}_\alpha \tag{1.29}$$

Biz magnit dipol va tashqi magnit maydon o‘rtasidagi magnit ta’sirini yetarlicha kuchsiz deb hisoblaymiz (tashqi sinov magnit maydonining kuchsizligi tufayli), shuning uchun $\left(D^{\mu\nu}F_{\mu\nu}\right)^2 = 0$ ifodadan foydalanishimiz mumkin. Ushbu tashqi magnit maydonda zarraning magnit impulse bu tashqi maydon bo‘ylab tekislanshishi kutilmoqda. Agar zarracha ekvatorial tekislikda harakat qilsa ($\theta = \frac{\pi}{2}$), unda bu magnit maydon faqat normal komponentaga ega $\hat{B}^\theta$. Bu ekvatorial tekislikda, xuddi shu narsa magnit dipolining eng kichik energiyasiga ega bo‘lgan holatga mos keladigan μ^θ magnit momenti uchun ham amal qiladi. Skalvar kattalik (1.29) dan keyin bo‘ladi:

$$D^{\mu\nu}F_{\mu\nu} = 2\mu B\mathcal{A} \tag{1.30}$$

Bu yerda $\mathcal{A}(r)$ butanosiblik funksiyasini belgilaydi, (1.10) tenglama va $\hat{B}^\theta$ uchun yoziladi:

$$\mathcal{A} = \mathcal{A}(r) = \sqrt{f}\frac{(1-\epsilon F_1)^{\frac{1}{2}}}{1-\epsilon F_2} \times \frac{1-\frac{\epsilon}{2}[rF_2' + 2F_2]}{\sqrt{1-\epsilon^2 F_1^2}} \tag{1.31}$$

$D^{\mu\nu}$ va $F^{\mu\nu}$ skalalar kattaliklarini (1.23) harakat tenglamasiga qo‘shib, ekvatorial tekislikda effektiv potensialni topish mumkin:

$$V_{\text{eff}} = \left(1 - \frac{2M}{r}\right)[1-\epsilon F_1] \times \left\{(1-\beta\mathcal{A})^2 + \frac{L^2}{r^2[1-\epsilon F_2]}\right\} \tag{1.32}$$

Bu yerda $\beta = 2\mu B/m$ magnit juftlik parametri deb ataladi; magnit dipol va tashqi magnit maydon o‘rtasidagi elektromagnit ta’sirini aniqlaydi. Haqiqiy astrofizik kuzatishlarda, masalan, magnit dipol moment $\mu = (1/2)B_{\text{NS}}R_{\text{NS}}^3$ bilan supermassive qora tuynuk atrofida aylanganida odatda neytron yulduzida magnit parametrlarga teng bo‘ladi:

$$\beta \approx \frac{11}{250}\left(\frac{B_{\text{NS}}}{10^{12}\,\text{G}}\right)\left(\frac{R_{\text{NS}}}{10^6\,\text{cm}}\right)^3\left(\frac{B_{\text{ext}}}{10\,\text{G}}\right)\left(\frac{m_{\text{NS}}}{M_\odot}\right)^{-1} \tag{1.33}$$

bu yerda B_{NS} — neytron yulduzining yuzasida magnit maydon, R_{NS} va m_{NS} — radius va neytron yulduz massasi. 5-rasm effektiv potensialning radiusga bog‘liqligi ko‘rsatilgan. Biz magnit dipolining effektiv potensial zaryadlangan zarrachalarning potensialiga o‘xshash tarzda harakat qilishini ko‘ramiz.

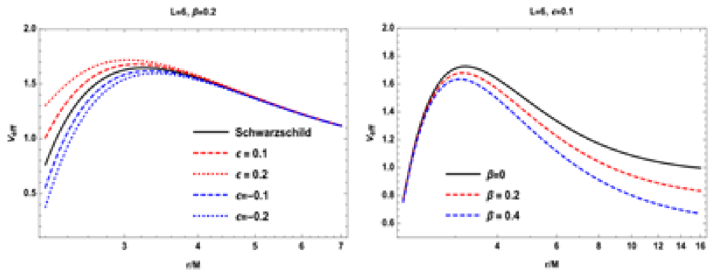

Figure 5. Radial dependence of the effective potential of the charged particle for various values of deformation and magnetic parameters.

Rasm 5. Deformatsiya va magnit parametrlarning turli qiymatlari uchun zaryadlangan zarracha effektiv potensialining radial bog’liqligi.

Zarrachaning aylanish traektoriyasi bilan bir xil sharoitda (1.21) dan foydalanib sinov zarrachasining impuls momenti va energiyasi uchun ifodalarni osongina toppish mumkin:

$$\mathcal{L}^2 = \frac{r^3(1-\alpha\beta)(1-\epsilon F_2)^2\{(1-\alpha\beta)\left[f_r(1-\epsilon F_1)-\epsilon f F_1'\right]-2\beta f \mathcal{A}_r(1-\epsilon F_1)\}}{(1-\epsilon F_2)\left[(1-\epsilon F_1)(sf-rf_r)+\epsilon f r F_1'\right]-fre(1-\epsilon F_1)F_2'} \tag{1.34}$$

$$\mathcal{E}^2 = \frac{(1-\beta\mathcal{A})f^2(1-\epsilon F_1)^2\{(1-\beta\mathcal{A})r\epsilon F_2'+2(1-\epsilon F_2)[1-\beta(\mathcal{A}+r\mathcal{A}_r)]\}}{(1-\epsilon F_2)\left[(1-\epsilon F_1)(2f-rf_r)+\epsilon f r F_1'\right]-fre(1-\epsilon F_1)F_2'} \tag{1.35}$$

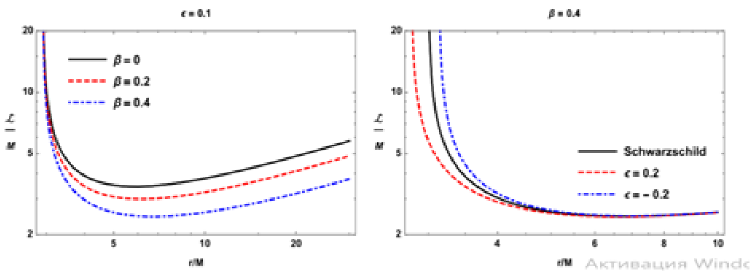

Figure 6. Dependence of the moment of momentum of the charged particle on the radius of the circular orbit at different values of the magnetic and deformation parameters.

Rasm 6. Magnit va deformatsiya parametrlarning turli qiymatlarida zaryadlangan zarra impuls momentining aylanma orbita radiusiga bog’liqligi.

6-rasmda ko’rsatilgandek ekvatorial tekislikda harakatlanadigan zarrachaning orbitasi, aylana, shuning uchun zarrachaning impuls momenti radial bog’liqlikka ega. Minimal chiziqlarni almashtirish orqali ISCO radiusning magnit parametrining turli qiymatlari, shuningdek, turli xil o’gish parametrlari uchun qanday o’zgarganligini aniqlash mumkin. Yuqori paneldan ko’rish mumkin, agar magnit dipol va tashqi magnit maydon o’rtasidagi magnit ta’sirni oshirsangiz, ISCO radiusi oshadi. Effektiv potensial va radial impuls momentiga bog’liqlik, endi ekvatorial tekislikda harakatlanadigan magnit dipol uchun ISCO tomonidan tekshirilishi mumkin. (1.22) tenglamada berilgan shartdan ham foydalanish mumkin yoki impuls moment kamida ISCO radiusiga ega bo’lishi kerak.

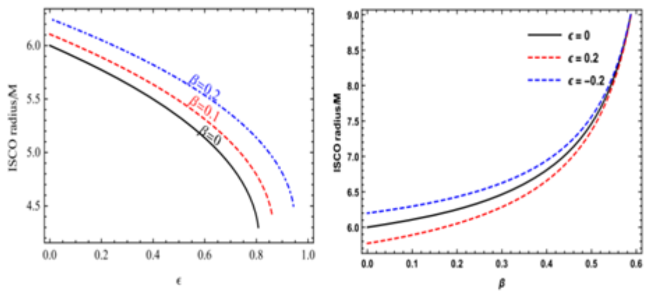

Figure 7. Around quasi-Schwarzschild black holes, certain test particles are the dependence of the Isco radius on the deviation parameter ϵ (left panel) and the magnetic parameter β (left panel) in the equatorial plane.
Rasm 7. Kvazi-Schwarzschildning qora tuynuklari atrofida ma'lum bir sinov zarrachalari ISCO radiusning ekvatorial tekislikda og'ish parametri ϵ (chap panel) va magnit parametr β (chap panel) ga bog'liqligi.

Keyin 7-rasmda ISCO radiusining ϵ parametrga va β magnit parametriga bog'liqligi ko'rsatilgan. 8-rasm magnit dipol ISCO radiusining qora tuynukning kvazi-Schwarzschild atrofida (yuqori panelda) va magnit (pastki panelda) parametrlarga bog'liqligini ko'rsatadi. Grafikdan ko'rinib turibdiki, magnit parametrining oshishi ISCO radiusini oshiradi. Bundan tashqari, ISCO radiusi cheksizlikka intilayotgan $\beta = \frac{2}{3}$ muhim ahamiyatga ega, magnit dipolining qanchaliktakrorlanishidan qat'iy nazar, barqaror aylanish orbitalar paydo bo'lmaydi. Yuqori panelda ISCO radiusning pastki qismini oldingi bobda bo'lgan kabi bir xil sabablarga ko'ra kesib tashladik, bu esa tanlangan parametrning burilish qiymati uchun sinovdan o'tgan zarrachaning impuls momenti kamida va maksimal darajada bo'lishi mumkinligini bildiradi, fizik jihatdan tegishli bo'lgan minimal nuqtalarni olishimiz kerak.

Xulosa

Biz maqolada zaryadlangan zarrachalarning harakati magnit dipolar bilan birgalikda fazo vaqt oralig'ida ϵ parametrining qanchalik yaxshi ekanligini aniqlash uchun tekshirildi va tashqi bir xil magnit maydonda Kerrning qora o'ra aylanish parametrini o'rnini bosishi mumkin, bu esa ushbu tadqiqotning asosiy yo'nalishi hisoblanadi. Zaryadlangan zarrachalarning harakatini o'rganish uni ko'rsatdiki, tashqi magnit maydon bo'lmasa og'ish parametri ϵ Kerr fazo-vaqtining aylanish parametrini $a \approx 0.46$ ga qadar o'rnini bosishi mumkin, bu esa Kerr qora o'rasi deb hisoblanadi, shuningdek aylanish parametric $\epsilon \approx 0.8$ og'ish parametrigacha qo'zg'almas kvazi-Schwarzschildning o'rnini ham bosishi mumkin. Bundan tashqari, tashqi magnit maydonning o'zi (ya'ni fazo-vaqt og'ish parametrisiz) aylanish parametrini $a \approx 0.88$ gacha aniqlik bilan taqlid qilishi mumkinligi ko'rsatilgan. Ushbu ikki parametrning kombinatsiyasi $a > 0.9$ gacha aylanish parametrini o'rnini bosishi mumkin. Magnit dipollarning dinamikasini o'rganish tashqi magnitdagi qora o'raning kvazi-Schwarzschild maydoni maksimal qiymati samarali ekanligini ko'rsatdi, magnit dipollarning singluar impuls momentining qiymatlari va qora o'ra atrofidagi fazo-vaqtning og'ish parametri magnit parametrining ortishi bilan ortadi. Musbat burilish parametric barqaror impuls moment qiymatlari va magnit juftlik parametrlarda effektiv potensialni oshiradi, manfiy qiymatlar esa kamayadi. $\epsilon_{\mathrm{cr}} > \epsilon \geq 0.35$ da sinovdan o'tgan zarrachalarning ISCO radiusining degeneratsiya qiymatlari mavjudligi ko'rsatildi, bu ikki xil ISCO radiusiga olib kelishi mumkin. Nihoyat, biz burilish parametri qora o'ra yad aylanishini qanday taqlid qilganini o'rganib chiqdik, bu esa sinov qilingan zarrachalarning ISCO radiusi uchun bir xil qiymatlarni taqdim etdi. Magnit dipollarni sinov sifatida ko'rib chiqsak, biz bu yerda $\beta \in [-0.25; 0.25]$ oralig'idagi magnit parametri uchun ikki xil belgilni tanladik. Natijada, og'ish parametrining $\epsilon \in (-1, 1)$ qiymati oralig'ida bo'lsa, magnit dipollar uchun $\frac{a}{M} \in (0.0537; 0.3952)$ oralig'ida aylangan yad qora o'ra aylanishini magnit parametri $\beta \in [-0.25; 0.25]$ qiymatlari aylangan orbitalic taqlid qilishi mumkin. Shu bilan birga, aylanivchi orbitalic aylanish parametrining simulyatsiya qiymatlari $\frac{a}{M} \in (0.0152; 0.78.63)$ oralig'ida joylashgan. Biz magnit parametri ISCO magnit dipollarining radiusini oshirganligi sababli, yadro qora o'ra aylanishini faqat aylangan orbitalarda $\beta = 0.5922$ qiymatiga tenglashtirish mumkinligini aniqladik.

Mualliflarning hissalari

X.M. metodologiya, dasturiy ta'minot, rasmiy tahlil, nazorat qilish, J.N. yozish va tahrirlash. Barcha mualliflar qo'lyozmaning nashr etilgan versiyasini o'qib chiqdilar va rozi bo'ldilar.

Authors' contribution

Conceptualization, Kh.M. methodology, software, formal analysis, supervision, J.N. writing and editing. All authors have read and agree with the published version of the manuscript.

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This study did not require ethical approval.

Nashrga xabardor qilingan rozilik

Barcha tadqiqotchilardan rozilik olindi

Consent for publication

Informed consent was obtained from all subjects involved in the study.

Ma'lumotlar mavjudligi to'g'risidagi bayonot

Ushbu maqolada keltirilgan ma'lumotlar mualliflarning ishi va manfaatdor shaxslar ushbu mavzu bo'yicha ma'lumot olish uchun yuqoridagi elektron pochta manzillariga murojaat qilishlari mumkin.

Data Availability Statement

The information presented in this article is the product of the authors' work, and those interested can contact the above-mentioned e-mail addresses regarding the information on the topic.

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Conflict of interest

The authors declare that there are no conflicts of interest in this research. All data and results were independently conducted during the research process, and there were no commercial or financial conflicts of interest between the researchers or institutions.

QisqartmalarQisqartmalar

Sgr	Sagittarius
SMQT	Successive Mean Quantization Transform
ISCO	International Standard Classification of Occupations

Adabiyot

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