



Article

# To'qnashuvchi zaryadlangan zarralarning massa markazi energiyasi

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## Xulosa:

Qora o'rالarni matematik jihatdan tavsiflashda ham umumiyl nisbiylik nazariyasi doirasidagi va alternativ gravitatsiya nazariyalaridagi turli yechimlar mavjud bo'lib bunday yechimlarni nazarij jihatdan sinov zarrachalarning qora o'ra atrofidagi harakatida o'zini qanday tutishiga qarab o'rganish mumkin. Qora o'ra atrofida magnit maydon mavjud deb qolaversa zarrachalar elektr zaryadiga yoki magnit xususiyatlarga ega deb qarash mumkin. Shu jihatdan bunday jarayonlarni o'rganish gravitatsiya nazariyalarini tekshirishda xususan qora o'ralar tabiatini o'rganishda muhim ahamiyat kasb etadi. Tashqi magnit maydonda joylashgan aksial-simmetrik qora o'ralar atrofidagi elektromagnit maydonlarni hamda zaryadli va sinov zarralar harakatida zarralar effektiv potensiallarining qanday o'zgarishini o'rganish va shu bilan bir qatorda Kerr-Taub-NUT metrikasi misoldida ko'rib chiqishdan iborat.

**Maqsad.** Qora o'ra atrofidagi zarralar harakatini o'rganish orqali kvazi-Schwarzschild va konform gravitatsiya sharoitidagi qora o'ra yechimlarini o'rganish.

**Materiallar va usullar.** Umumiy nisbiylik nazariyasida makroskopik elektrodinamikaning matematik apparati; zarra va maydonning harakat tenglamalarini yechishning analitik va raqamli usullari. Yechimlarda qatnashadigan fazo-vaqt parametrlarining zarralar harakatiga ta'sirini o'rganish; bunday yechimlar umumiyl nisbiylik nazariyasdagi yechimlardan qanchalik farq qilishi yoki ularning effektlarini namoyon qila olish qobiliyatini o'rganish; tashqi magnit maydonning mavjudligi masalaga qanchalik hissa qo'shishini baholash.

**Natijalar.** Kvazi-Schwarzschild va conformgravitatsiya sharoitidagi qora o'ralar tashqi magnit maydoniga kiritilgan hollar uchun zarralar harakati birinchi marta o'rganildi va bu yechimlar qaralgan effektlarning umumiyl nisbiylik nazariyasdagi yechimlardan qay darajada farqlanishi ko'rib chiqildi. Bunda tashqi elektromagnit maydonning bu effektlarga ta'siri qaralgan yechimlar uchun birinchi marta baholandi. Kvazi-Schwarzschild yechimning deformatsiya parametri tashqi magnit maydon bo'lgan va bo'lмаган hol uchun umumiyl nisbiylik nazariyasdagi Kerr yechimining spin parametrini qanchalik o'rnini bosishi baholandi.

**Xulosa.** Magnit parametri va ISCO radius qiymatlari uchun og'ish parametrlari o'rtasidagi degeneratsiya munosabatlарини ko'rsatdi va magnit parametrining qiymatlari uchun inkor etish parametrining ikki xil musbat qiymatida ISCO radiusining bir xil bo'lishi mumkinligini aniqladik. Tadqiqot SMQT ga yaqin muhitda magnitlangan modda va neytron yulduzlarining dinamikasiga qo'llanilishi mumkin. Magnit dipollarning dinamikasini o'rganish tashqi magnitdagagi qora o'raning kvazi-Schwarzschild maydoni maksimal qiymati samarali ekanligini ko'rsatdi, magnit dipollarning singular impuls momentining qiymatlari va qora o'ra atrofidagi fazo-vaqtning og'ish parametri magnit parametrining ortishi bilan ortadi.

**Kalit so'zlar:** qora o'ra, kvazi-Schwarzschild, konform gravitatsiya, spin, elektromagnit, effektiv potensial.

## The center-of-mass energy of colliding charged particles

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### Abstract:

In the mathematical characterization of black holes, there are various solutions both within the framework of general relativity and in alternative theories of gravity. These solutions can be theoretically studied by analyzing the behavior of test particles moving around black holes. If a magnetic field exists around a black hole, and if the particles possess electric charge or magnetic properties, such processes become crucial in testing gravity theories and understanding the nature of black holes. This research focuses on the electromagnetic fields and the motion of charged and test particles around axially symmetric black holes located in an external magnetic field. It also studies how the effective potentials of particles change, particularly through the example of the Kerr–Taub–NUT metric.

**Background.** To explore black hole solutions in quasi-Schwarzschild and conformal gravity settings by studying the motion of particles around black holes.

**Materials and methods.** The mathematical framework of macroscopic electrodynamics in general relativity; analytical and numerical methods for solving the equations of motion for particles and fields; analysis of the influence of spacetime parameters on particle motion; examination of how these solutions differ from those in general relativity and their ability to reveal new effects; assessment of the contribution of an external magnetic field to the problem.

**Results.** For the first time, the motion of particles in external magnetic fields around black holes in quasi-Schwarzschild and conformal gravity contexts has been studied. The extent to which these solutions differ from those in general relativity has been analyzed. The influence of the external electromagnetic field on these effects was evaluated for the first time for the considered solutions. The deformation parameter of the quasi-Schwarzschild solution was assessed for both the presence and absence of an external magnetic field, in terms of how well it substitutes the spin parameter in the Kerr solution of general relativity.

**Conclusion.** We demonstrated the degeneracy relations between the magnetic parameter and the ISCO radius values and found that for two different positive values of the deviation parameter, the ISCO radius can be the same for specific magnetic parameter values. The study can be applied to the dynamics of magnetized matter and neutron stars in environments close to SMQT. The investigation of magnetic dipole dynamics showed that the maximum value of the quasi-Schwarzschild field of a black hole in an external magnetic field is effective. The values of the singular angular momentum of magnetic dipoles and the deviation parameter of spacetime around the black hole increase with the magnetic parameter.

**Keywords:** black hole, quasi-Schwarzschild, conformal gravity, spin, electromagnetic, effective potential

### Kirish

Hozirgi vaqtida neytron yulduzlarini aniqlashga urinishlariga qaramasdan somon yo'li markazida gravitatsion hamkorlik orqali ultra massivli qora tuynuk Sagitarus A\* yoy yaqinidagi qayta ishlangan radio modullari sifatida, ularning astrofizik kuzatuvlar yo'q. SgrA\* atrofida pulsarlarning yo'qlining sabablaridan biri SMQT atrofidagi plazma muhitida radio to'lqinlarning tarqalishi, ikkinchisi esa neytron yulduzining dipol momenti va markaziy qora o'ra magnit zaryadlar yoki elektr toki natijasida hosil bo'lgan qora o'ra atrofidagi magnit maydon o'rtasidagi magnit maydonning ta'siri bo'lishi mumkin. Neytral zarrachalarning barqaror aylanma va xaotik harakati, statik va aylanadigan qora o'ralar atrofida zaryadlangan zarrachalarning dinamikasi va kvazigarmonik tebranishlari tashqi asinxron bir xil magnit maydonlarga va plazma magnitosferasi turli xil qora o'ra larni o'rab oladi. Xususan, Lyapunov usuli yordamida muntazam va xaotik orbitalar o'rtasidagi farqni ko'rsatish mumkin. Bundan tashqari, kichik bir tizimsiz va tortishish effektlari xaotik harakatining pasayishiga olib keladi.

Ushbu ishda bizning asosiy maqsadimiz magnitlangan kvazi-Schwarzschild qora o'ra atrofida zaryadlangan zarralar va magnit dipollarning harkatini o'rganishdir. Ish quyidagicha tashkil etilgan:

qora o'raning atrofida zaryadlangan zarrachalarning dinamikasini va Kerr fazo vaqtiga nisbatan o'rganishga bag'ishlangan. Magnit dipolning tashqi magnit maydonga tushirilgan qora o'raning kvazi-Schwarzschild atrofida harakatlanishi o'rganiladi. Ushbu bo'limda olingen natijalar Kerr qora o'ra atrofidagi zarrachalarning dinamikasi bilan taqqoslanadi. Ish davomida biz  $G=1=c$  bo'lган bo'shliqqa o'xshash belgi  $(-, +, +, +)$  va birlik tizimdan foydalanmiz.

### Natijalar

Ushbu bobda biz jonform gravitatsiya sharoitida aylanadigan magnitlangan qora tuynuklar yaqinidagi ikkita zaryadlangan zarrachalarning to'qnashuvi massa markazining energiyasini o'rganamiz.  $m_1$  va  $m_2$  massalari va  $u_1^\alpha$  va  $u_2^\alpha$  ning 4-tezligi balanssizlikdan keladigan ikkita zarrachalar massa markazining energiyasi uchun ularning 4-impulsining umumiy ifoda,

$$\{E_{cm}, 0, 0, 0\} = m_1 u_1^\mu + m_2 u_2^\mu \quad (1.1)$$

Massalar markazining energiya maydoni 1.1 dan aniqlanishi mumkin va bizda

$$E_{cm}^2 = m_1^2 + m_2^2 - 2m_1 m_2 g_{\mu\nu} u_1^\mu u_2^\nu \quad (1.2)$$

Algebraic o'zgarishlardan so'ng, biz o'lchamsiz shaklda ifoda etamiz

$$\frac{E_{cm}^2}{m_1 m_2} = \frac{m_1}{m_2} + \frac{m_2}{m_1} - 2g_{\mu\nu} u_1^\mu u_2^\nu \quad (1.3)$$

4-tezlik uchun ifodani ishlatalish tegishli tortishish bilan magnitlangan aylanuvchi qora tuynuklar atrofidagi zaryadlangan zarralar massalari teng  $m_1=m_2$  va  $E_1=E_2=1$  boshlang'ich energiyasi bilan to'qnashadi deb hisobga olingen holda massa markazining energiyasi uchun quyidagi shaklda ifodalanishi mumkin

$$\frac{E_{cm}^2}{2m^2} = E_{cm}^2 = 1 - g_{\mu\nu} u_1^\mu u_2^\nu \quad (1.4)$$

Magnit parametri  $|\omega_B| = 0.1$  bilan musbat va musbat (musbat va manfiy) zaryadlangan zarrachalarning to'qnashuv markazining energiyasini tahlil qilamiz,  $E_1 = E_2 = 1$  boshlang'ich energiyasi va impuls momenti  $L_1 = -L = 4$  bo'lган cheksizlikdan kelib chiqadi. 1- va 2-rasm tashqi magnit maydonda aylanadigan qora tuynuk yaqinidagi musbat-musbat va musbat-manfiy zaryadlangan zarrachalar to'qnashuv massalari markazining *energiyasining radiusga bog'liqligi* ko'rsatilgan. Ramslardan ko'rinish turibdiki, *massa markazining energiyasi l* conform parametri va aylanish parametrini oshirish bilan kamayadi. Bundan tashqari, musbat zaryadlangan zarralar to'qnaashuvlarida massa markazining enrgiyasi Kulon kuchlarining ustunligi tufayli yo'qoladi.

**Table 1.** The innermost stable circular orbits of particles moving around a rotating black hole.

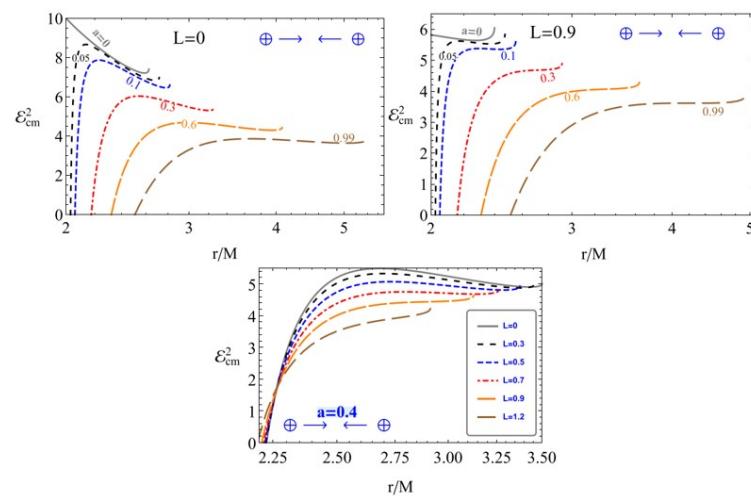
**Jadval 1.** Aylanadigan qora tuynuk atrofida harakatlanadigan zarrachalarning eng ichki barqaror aylanma orbitalari.

$L^2$	0	0.001	0.002	0.005	0.01
$\omega_B = 0.05$	3.7312	3.7314	3.7316	3.7322	3.7331
$\omega_B = 0.1$	3.2897	3.2899	3.29	3.2904	3.2911
$\omega_B = 0.2$	2.8444	2.8450	2.8446	2.8449	2.8504
$\omega_B = 0.4$	2.4799	2.48	2.4801	2.4803	2.4807
$\omega_B = 0.8$	2.2192	2.2193	2.2194	2.2150	2.2197

**Table 2.** The innermost stable circular orbits of particles moving around a rotating black hole ( $\omega_B=0.25$  and  $M=1$ )

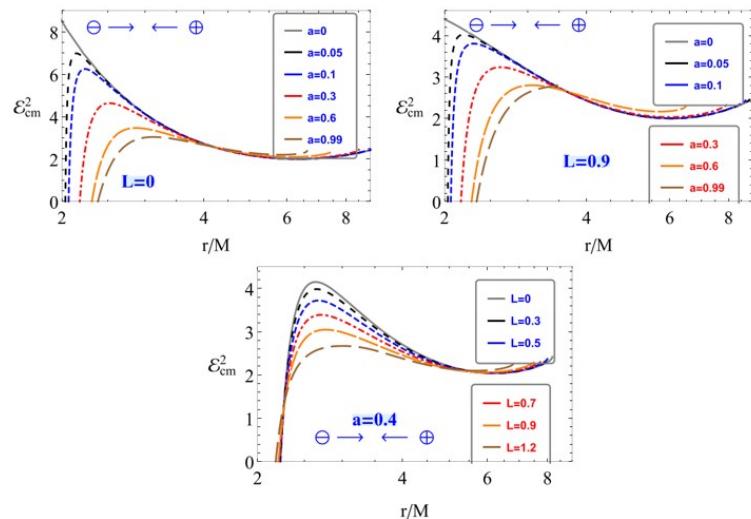
**Jadval 2.** Aylanadigan qora tuynuk atrofida harakatlanadigan zarrachalarning eng ichki barqaror aylanma orbitalari ( $\omega_B=0.25$  va  $M=1$  uchun).

$L^2$	0	0.001	0.002	0.005	0.01
$a = 0.05$	3.1458	3.1459	3.146	3.1462	3.1467
$a = 0.1$	3.1074	3.1075	3.1076	3.1078	3.1083
$a = 0.2$	2.0348	3.0249	2.025	2.0253	2.0257
$a = 0.4$	2.8316	2.83165	2.8317	2.832	2.8324
$a = 0.8$	2.2265	2.2267	2.2268	2.2273	2.2281
$a = 0.99$	1.4045	1.4049	1.4054	1.4067	1.4092



**Figure 1.** The dependence of the center-of-mass energy of two colliding particles on the radius, in the case of conformal gravity, for positively charged particles near a black hole with different values of the conformal parameter  $L$  and rotation parameter. In units where  $M=1$ .

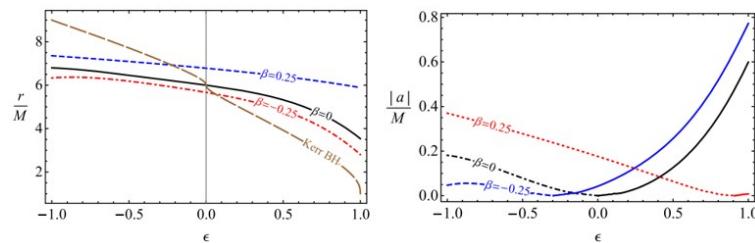
**Rasm 1.** Ikki to'qnashuv zarra massa markazi energiyasining radiusga bog'liqligi, konform gravitatsiyasi sharoitida qora tuynuk yaqinidagi musbat zaryadlangan zarrachalarning turli konform parametr  $L$  qiyatlari va aylanish parametri bo'lgan hol uchun.  $M=1$  bo'lgan o'lchov birliklarda.



**Figure 2.** As in Fig. 1, but for collisions of positively and negatively charged particles. In units where  $M=1$

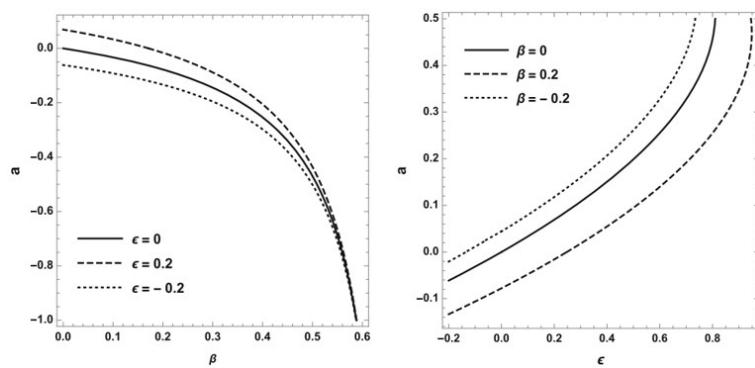
**Rasm 2.** 1-rasm kabi, lekin musbat va manfiy zaryadlangan zarrachalarning to'qnashuvlari uchun.  $M=1$  bo'lgan o'lchov birliklarda.

**Magnit momentli zarralar harakatining astrofizik qo'llanilishi** Relativistik astrofizikaning muhim va dolzorb masalalaridan biri test magnit dipolining harakatini, xususan, neytron yulduzlarining (pulsar yoki magnetarlarning) harakatini o'rganish orqali tortishish nazariyasini tekshirishdir. SMQT atrofidagi tortishish va elektronnit maydonlarni kuzatilshardagi aniq impulslar tufayli tekshirishga imkon beradigan supermassiv qora tuynuklar (bundan keyin SMQT) atrofida test magnit dipollari sifatida qaraladi. Doppler effekti bilan massoni o'chashga yordam berishi mumkin. Bunday modellarni kuzatishda neytron yulduzining galaktik markaz yaqinida topilishi mumkinligini tushunish mumkin. Biroq, bugungi kunda Sgr A\* atrofida zinch zaryadlangan electron gazda radio modullarini topish juda qiyin. Birinchi va hozirgi vaqtida yagona neytron yulduzi magnetar SGR 1745-2900 nomi bilan Sgr A\* atrofida 2013 yilda topilgan [6]. Hisob-kitoblarimizda magnit parametrlerini uni Sgr A\* atrofida aylanadigan magnit dipol deb hisoblashdan foydalanamiz. Boshqa tomondan, nazariy munozaralarda ISCO radiusi SMQT atrofida kuzatilgan xususiyatlarni tahlil qilishda, turli xil tortishish o'zgaruvchilari parametrleri xususiyatlarga o'xshash ta'sirlarni aks ettirganda paydo bo'ladi; bunday hollarda tortishish effekti dominant rol o'ynashi mumkin emas. Aslida, asosan astrofizik qora tuynuklar aylanadigan qora tuynuklar sifatida olinadi. Bu yerda biz ISCO radiusini fazoviy deformatsiyasi va Kerr qora tuynugining aylanishi bilan solishtirib, magnit dipollarining ISCO radiusining bir xil qiymatini ta'minlagan holda tahlil qildik. Taqqoslash uchun, biz tashqi asimptotik bir xil magnit maydonga va magnit maydonsiz Kerr qora tuynugiga kiritilgan kvazishvartschild kompakt obyektni ko'rib chiqdik. Biz SMQT Sgr A\* atrofida aylanadigan magnetarning haqiqiy astrofizik holatini taxmin qilamiz. Shuni ham ta'kidlash kerakki, magnit dipol tashqi magnit maydon bo'lmasa neytral deb hisoblanishi mumkin. O'ta massiv qora tuynuk Sgr A\* atrofida aylanadigan dipol momenti  $\mu \simeq 1.6 \times 10^{32}$  G bo'lgan SGR (PSR) J1745-2900 magnetari uchun *magnit juftlik parametri* [1].



**Figure 3.** Left panel: Dependence of the ISCO radius on the deformation parameter for a magnetic dipole. Right panel: Degeneracy between the spin parameter of the Kerr solution and the deformation parameters of the quasi-Schwarzschild solution that yield the same ISCO radius.  $\beta = 0.25, 0, -0.25$ .

**Rasm 3.** Chap panel: magnit dipol uchun ISCO radiusning deformatsiya parametriga bog'liqligi. O'ng panel: Bir xil ISCO radiusini beradigan hol uchun Kerr yechimi spin parametri va kvazi-Schwarzschild yechimi deformatsiya parametrleri orasidagi degeneratsiya.  $\beta = 0.25, 0, -0.25$ .



**Figure 4.** Left panel: Degeneracy between the spin parameter of the Kerr solution and the deformation parameter of the quasi-Schwarzschild solution that yield the same ISCO radius.

**Rasm 4.** Chap panel: bir xil ISCO radiusini beradigan hol uchun Kerr yechimi spin parametri va kvazi-Schwarzschild yechimi deformatsiya parametri orasidagi degeneratsiya.

$$\beta_{\text{PSR J1745-2900}} \simeq 0.716 \left( \frac{B_{\text{ext}}}{10G} \right) \quad (1.5)$$

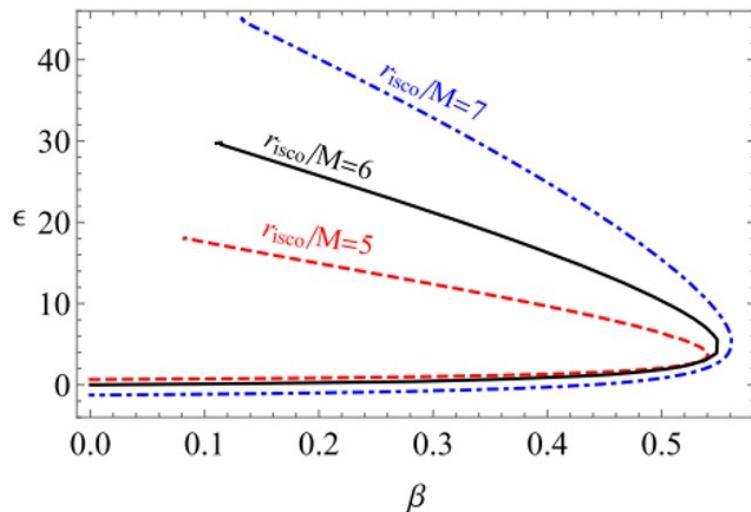
Burilish atrofida ISCO sinov zarralar radiusi Kerrning qora tuynuklari retrograde (+) va progressive (-) orbitalar uchun quyidagi ifoda bilan belgilanadi:

$$r_{\text{isco}} = 3 + Z_2 \pm \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)} \quad (1.6)$$

Bu yerda

$$Z_1 - 1 = \left( \sqrt[3]{1+a} + \sqrt[3]{1-a} \right) \sqrt[3]{1-a^2}, \quad Z_2^2 - Z_1^2 = 3a^2 \quad (1)$$

Endi, ISCO radiusiga aylanish va burilish parametrlarining ta'sirini solishtirish uchun biz quyidagilarni taqdim etamiz. ISCO radiusining magnit dipollari uchun magnit aloqa parametrlarining manfiy va musbat qiymatlari,  $\beta = \pm 0.25$  va neytral zarralar bilan Kerr qora teshigining burilish va aylanish parametriga bog'liqidir. O'rganilayotgan zarrachaning magnit dipol momentining yo'naliishi magnit maydonning yo'anilishi bilan bir xil bo'lsa, magnit aloqa parametri musbat bo'ladi, aks holda u manfiy. 3-shakl bo'yicha biz tashqi magnit maydonning mavjudligi va yo'qligi (ko'k nuqta, qizil nuqtali va qora rangli chiziqlar) va tashqi magnit maydonning yo'qligida va tashqi magnit maydonning yo'qligida aylanadigan Kerr qora tuynugi atrofida ISCO magnit dipol radiusining xattiharakatlarini taqdim etamiz. Ba'zi hollarda, bu Kerrning qora tuynukining aylanish ta'siriga o'xshaydi. Bundan tashqari, magnit parametrining musbat (manfiy) qiymatlari ISCO radiusini markaziy qora tuynukdan (yon tomonga) o'zgartiradi. Pastki panelda biz ISCO radiusi uchun bir xil qiymatlarga ega bo'lgan magnit dipollarning asl holatlari uchun kvaziSchwarzschild og'ish parametrining va Kerr qora tuynugining aylanishini (tag'oslashni) ko'rsatamiz. Degeneratsion grafigidan ko'rinish turibdiki, manfiy og'ish parametri yadroviy qora tuynukning aylanishini taqlid qilishi mumkin. Bu magnitlangan zarrachalarning aylanadigan orbitalari uchun magnit aloqa parametri  $\beta = 0.25$  dan  $\frac{a}{M} \simeq 0.3952$  ga teng bo'lgan radius qiymatini ta'minlaydi,  $\beta = 0$  va  $\beta = -0.25$  parametrlari zarralar uchun  $a \simeq 0.1984M$  va  $a \simeq 0.0537M$  qora yadro teshigining aylanish qiymatiga mos keladi. Bu yerda biz magnit parametri Kerr qora tuynugining aylanishini simulyatsiya qila olamiz, Schwarzschild qora tuynuk atrofida magnit dipolar uchun ISCO radiusini osongina hisoblash mumkin, bu esa og'ish parametrini nolga tenglashtiradi.



**Figure 5.** The relationship between the deformation parameter  $\beta$  for the values of the ISCO radius of a magnetic dipole.

**Rasm 5.** Magnit dipolar ISCO radiisuining qiymatlari uchun og'ish parametri va  $\beta$  magnit parametri o'rtaisdagi munosabatlari.

Schwarzschild metrikasi va Kerr aylanish parametrinda magnit parametr o'rtaida degeneratsiya grafikasini chizishimiz mumkin. O'ng paneldag'i 5-rasm burilish parametrining qiymatlari bilan,

ISCO radiusi magnit parametri ortib borayotgan holatda ham shunga o'xshash tarzda o'sadi. Bu esa, bu parametri faqat retrograd orbitalar uchun Kerr metrikasining aylanish parametrini o'rnini bosishi kerak degan xulosaga olib keladi. Bu 4-rasmida ko'rsatilgandek, degeneratsiya grafigida ko'rish mumkin. 5-rasmida Parametr qiymatlari va magnit parametrlari o'rtasida ISCO radiusining qiymatlari mavjudligini ko'rish oson bo'ladi. Shuning uchun, ISCO radiusining ortishi bilan og'ish parametrining degeneratsiya qiymatlari oralig'I oshadi.

### Xulosa

Magnit parametri va ISCO radius qiymatlari uchun og'ish parametrlari o'rtasidagi degeneratsiya munosabatlarini ko'rsatdik va magnit parametrining qiymatlari uchun inkor etish parametrining ikki xil musbat qiymatida ISCO radiusining bir xil bo'lishi mumkinligini aniqladik. Tadqiqot SMQT ga yaqin muhitda magnitlangan modda va neytron yulduzlarning dinamikasiga qo'llanilishi mumkin. Magnit dipollarining dinamikasini o'rganish tashqi magnitdag'i qora o'raning kvazi-Schwarzschild maydoni maksimal qiymati samarali ekanligini ko'rsatdi, magnit dipollarining singular impuls momentining qiymatlari va qora o'rta atrofidagi fazo-vaqtning og'ish parametri magnit parametrining ortishi bilan ortadi. Musbat burilish parametri barqaror impuls moment qiymatlari va magnit juftlik parametrlarida effektiv potentsialni oshiradi, manfiy qiymatlar esa kamayadi.  $\epsilon_{cr} > \epsilon \geq 0.35$  da sinovdan o'tgan zarrachalarning ISCO radiusining degeneratsiya qiymatlari mavjudligi ko'rsatildi, bu ikki xil ISCO radiusiga olib kelishi mumkin. Nihoyat, biz burilish parametri qora o'rta yadroviy aylanishi qanday taqlid qilinishini o'rgandik, bu esa sinov qilingan zarrachalarning ISCO radiusi uchun bir xil qiymatlarni taqdim etdi. Magnit dipollarni sinov sifatida ko'rib chiqsak, biz bu yerda  $\beta \in [-0.25; 0.25]$  oralig'idagi magnit parametri uchun ikki xil belgi tanladik.

### Mualliflarning hissaları

Konseptualizatsiya, metodologiya, dasturiy ta'minot, rasmiy tahlil, nazorat qilish, Yozish va tahrirlash X.M. Barcha mualliflar qo'lyozmaning nashr etilgan versiyasini o'qib chiqdilar va rozi bo'ldilar.

### Authors' contribution.

Conceptualization, methodology, software, formal analysis, supervision, writing and editing X.M. All authors have read and agreed to the published version of the manuscript.

### Moliyalashtirish

Ushbu tadqiqot xech qanday moliyalashtirilmagan

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### Etika tamoyillariga muvofiqlik

Ushbu tadqiqot meyorlar asosida.

### Ethics approval.

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### Nashrga xabardor qilingan rozilik.

Barcha tadqiqotchilardan rozilik olindi

### Consent for publication.

Informed consent was obtained from all subjects involved in the study.

### Ma'lumotlar mavjudligi to'g'risidagi bayonot

Ushbu maqolada keltirilgan ma'lumotlar mualliflarning ishi va manfaatdor shaxslar ushbu mavzu bo'yicha ma'lumot olish uchun yuqorida elektron pochta manzillariga murojaat qilishlari mumkin.

### Data Availability Statement

The information presented in this article is the product of the authors' work, and those interested can contact the above-mentioned e-mail addresses regarding the information on the topic.

### Rahmatnomalar

Ushbu ilmiy tadqiqotni amalga oshirish jarayonida bevosita mualliflik hissasi yoki moliyaviy qo'llab-quvvatlash bilan bog'liq bo'lмаган quyidagi ko'maklar uchun minnatdorchilik bildiramiz. Jumladan, ma'muriy va texnik yordam ko'rsatgan hamda tadqiqotda foydalanilgan materiallar bilan amaliy yordam bergen barcha hamkasblarimizga va muassasalarga chuqrur minnatdorchilik izhor etamiz.

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Mualliflar ushbu tadqiqot natijalarini taqdim etish yoki talqin qilishga nomaqbul ta'sir ko'rsatishi mumkin bo'lgan hech qanday shaxsiy holat yoki manfaatlarga ega emasliklarini ma'lum qiladilar. Tadqiqotni loyihalashda, ma'lumotlarni yig'ish, tahlil qilish, sharhlashda, qo'lyozmani yozishda yoki natijalarni nashr etish bo'yicha qaror qabul qilish jarayonida hech qanday homiylar ishtirot etmagan

### Conflict of interest

The authors declare that they have no personal circumstances or interests that could inappropriately influence the presentation or interpretation of the research results. No sponsors were involved in the design of the study; in the collection, analysis, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

### Qisqartmalar

SMQT	Supermassiv qora tuyruk
ISCO	Eng ichki barqaror aylanma orbita
G	Gravitatsion doimiylik ( $G = 1$ birlik tizimida)
c	Yorug'lik tezligi ( $c = 1$ birlik tizimida)
M	Qora o'ra massasi ( $M = 1$ birlikda normallashtirilgan)
a	Kerr qora tuynugidagi aylanish parametri
$\beta$	Magnit aloqa (birikma) parametri
$\omega_B$	Magnit parametri (tashqi magnit maydon kuchi bilan bog'liq)
$E_{cm}$	To'qnashuvdagи massa markazi energiyasi
L	Impuls momenti
$u^n$	To'rt-tezlik komponentlari
$g_{\mu\nu}$	Fazo-vaqt metrik tensorining komponentlari

### Adabiyot

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