

Article

Diskret pt-simmetrik panjaralar: spektral xususiyatlar va solitonlarning stabiligi

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Xulosa:

Ushbu ishda diskret PT-simmetrik panjara va halqa ko'rinishidagi panjaralarning spektral xususiyatlari va nochizikli holdagi PT-simmetrik solitonlarning chiziqli stabiligi tahlil qilinadi. Bunda PT-simmetrik potensial haqiqiy qismi nolga teng va mavhum qismi esa har bir nuqta uchun musbat va manfiy qiymatlar qabul qiluvchi Gauss potentsiali ko'rinishida beriladi. Natijalar shuni ko'rsatadiki, PT-simmetriya buzilish nuqtasida tizimda fazaviy o'tish yuz beradi va bu energiya spektrining haqiqiydan kompleks qiymatlarga o'tishi bilan tavsiflanadi. Nochizikli holda PT-simmetriya buzilmagan holda chiziqli stabil, buzilgan holda esa chiziqli nostabil ekani ko'rsatildi.

Kalit so'zlar: PT-simmetriya, optik panjara, diskret Schrödinger tenglamasi.

Discrete pt-symmetric lattices: spectral properties and stability of solitons

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Abstract:

In this work, the spectral properties of discrete PT-symmetric lattices and ring-shaped lattices, as well as the linear stability of PT-symmetric solitons in the nonlinear regime, are investigated. The PT-symmetric potential is considered such that its real part is equal to zero, while its imaginary part is represented by a Gaussian potential taking positive and negative values at each lattice site. The obtained results show that a phase transition occurs in the system at the PT-symmetry breaking point, which is characterized by the transition of the energy spectrum from real to complex values. Furthermore, it is shown that in the nonlinear regime, solitons are linearly stable in the unbroken PT-symmetric phase, whereas they become linearly unstable in the broken PT-symmetric phase.

Keywords: PT-symmetry, optical lattice, discrete Schrödinger equation, soliton, linear stability.

Kirish

So'nggi yillarda PT-simmetrik tizimlar fizikaning turli sohalarida, xususan, kvant mexanikasi, optika va to'liq dinamikasida muhim tadqiqot yo'nalishiga aylandi [1,15]. An'anaviy kvant mexanikasida ermit operatorlar faqat haqiqiy energiya spektrini beradi, biroq PT-simmetrik ermit bo'lmagan operatorlar ham ma'lum shartlarda to'liq haqiqiy xos qiymatlar spektriga ega bo'lishi mumkin. Bu holat PT-simmetriyaning buzilmagan fazasi deb ataladi va tizim parametrlarining o'zgarishi bilan PT-simmetriyaning buzilish nuqtasida fazaviy o'tish ro'y beradi, bunda haqiqiy energiya spektri kompleks qiymatlarga o'tadi [16,23].

Kvant mexanikadan tashqari, PT-simmetrik tizimlar turli yo'nalishlarda — topologik fazalardan biologik tizimlargacha — keng o'rganilgan. Ularning ichida eng ko'p tadqiq qilingan

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sohalardan biri optik/fotonik tizimlardir (qarang, masalan, [2–4]). Chiziqli optik muhitlarda PT-simmetriya fazoviy simmetrik sindirish ko‘rsatkichi hamda muvozanatlashgan kuchayish–susayish qismlariga ega tuzilmalarda namoyon bo‘ladi. PT-simmetriya buzilganda, yorug‘lik nuri intensivligi eksponensial ravishda ortadi yoki kamayadi. Ushbu xususiyat yorug‘lik orqali boshqariladigan qurilmalarni yaratish imkonini beradi [5].

Mazkur tadqiqotlarda [6–8] PT-simmetrik optik panjaralarning dispersion munosabatlari o‘rganilgan bo‘lib, turli kuchayish–susayish modulyatsiyalari qanday qilib turli spektral xususiyatlar va PT-simmetriyaning buzilish holatlarini yuzaga keltirishi ko‘rsatib berilgan. PT-simmetrik optik tizimlarning ilk eksperimental realizatsiyasi ikki kanal orasidagi passiv [9] va aktiv [10] PT-simmetriya buzilishini yaratish orqali amalga oshirilgan. Sun‘iy fotonik panjaralar yordamida bir yo‘nalishli “ko‘rinmas” muhit hosil qilingan [11].

O‘zaro bog‘langan ikkita mikrokovak rezonator PT-simmetriyaning buzilishini namoyish qilishi mumkin bo‘lib, nohiziqli effektlar yangi avlod PT-simmetrik optik tizimlarida noetsiprok (bir tomonlama) xususiyatlarni ta‘minlaydi [12]. Shuningdek, optik izolyatsiya [13], faza o‘tishlari [14] va topologik chekka holatlar [15] mos ravishda mikrorezonatorlar, atom panjaralari va kvant yurishlarida eksperimental ravishda kuzatilgan.

Uzluksiz muhitlardan farqli ravishda, diskret panjarali tizimlar PT-simmetrik effektlarni nazorat qilish va ularni eksperimental amalga oshirish uchun qulay platforma hisoblanadi. Ayniqsa, nohiziqli PT-simmetrik diskret tizimlar lokalizatsiyalangan to‘lqin rejimlari – solitonlarning mavjudligi va ularning barqarorligini o‘rganishda muhim ahamiyat kasb etadi.

Mazkur ishda PT-simmetrik diskret nohiziqli Schrödinger tenglamasi asosida bir o‘lchamli panjara va halqa ko‘rinishidagi panjara tizimlarida solitonlarning hosil bo‘lishi va ularning chiziqli barqarorligi nazariy jihatdan tahlil qilinadi. Asosiy e‘tibor PT-simmetriya buzilish nuqtasining sistemaning geometrik parametrlariga (masalan, kuchayish-susayish profilining fazoviy kengligi) bog‘liqligi va bu bog‘liqlikning soliton dinamikasiga ta‘siriga qaratilgan.

PT-simmetrik nohiziqli Schrödinger tenglamasi

2.1. Konotop modeli

Ushbu paragrafda biz qisqacha PT-simmetrik diskret nohiziqli Schrödinger (DNS) tenglamasini quyidagi tarzda keltiramiz [24]

$$i\frac{dq_n}{dt} + c_n(q_{n+1} - q_n) + c_{n+1}(q_{n-1} - q_n) - g|q_n|^2q_n + i(-1)^{n+1}\gamma q_n = 0, \quad (1)$$

bu yerda $q_n - n$ -tugundagi maydon amplitudasi, $c_n - qo‘shni$ tugunlar orasidagi bog‘lanish koeffitsienti ($n = 2p + 1$ uchun $c_n = \kappa_1$, $n = 2p$ uchun $c_n = \kappa_0$), $g - nohiziqilik$ koeffitsienti, $\gamma - kuchayish-susayish$ kuchi, $i(-1)^{n+1}\gamma - PT$ -simmetrik potensial.

Chiziqli hol ($g = 0$) uchun PT-simmetriya buzilmaslik sharti quyidagicha yoziladi [24]:

$$|\kappa_0 - \kappa_1| \geq \gamma. \quad (2)$$

Yoki yuqoridagi shartni quyidagicha yozishimiz mumkin:

$$(\kappa_0 - \kappa_1)^2 - \gamma^2 \leq \mu^2 \leq (\kappa_0 + \kappa_1)^2 - \gamma^2. \quad (3)$$

(2) shart xos qiymatning xususiyatini belgilaydi. Agar bu shart qanoatlantirilmasa, keyin xos qiymatning mavhum qismi noldan farqli bo‘ladi.

Endi nuqtalarni kuchayuvchi u_n va susayuvchi v_n nuqtalarga ajratamiz va o‘zgaruvchilarni ajratish usulidan foydalanamiz:

$$\begin{pmatrix} q_{2n}(t) \\ q_{2n+1}(t) \end{pmatrix} = \begin{pmatrix} u_n \\ v_n \end{pmatrix} e^{-i(\kappa_0 + \kappa_1 + \mu)t}, \quad (4)$$

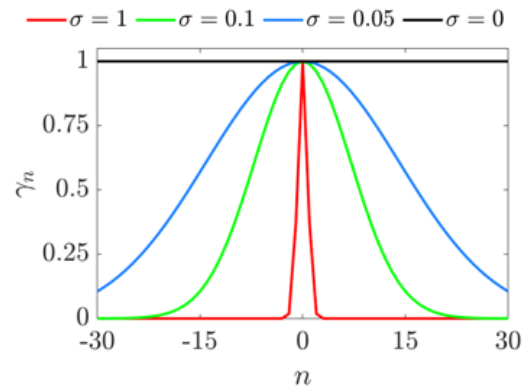


Figure 1. The spatially localized gain–loss profile in Eq. (8). Here, $\gamma_0 = 1$ was chosen.

Rasm 1. (8)-tenglamadagi fazoda lokalizatsiyalashgan kuchayish-susayish profili. Bu yerda $\gamma_0 = 1$ deb tanlab olindi.

Bu yerda u_n va v_n quyidagi asimptotik chegara shartni qanoatlantiradi:

$$\begin{pmatrix} u_n \\ v_n \end{pmatrix}_{n \rightarrow \pm\infty} \rightarrow 0.$$

DNS tenglama (1) o‘zaro bog‘langan nochiqliq algebraik tenglamalar sistemasi sifatida quyidagicha qayta yozilishi mumkin:

$$\begin{cases} (\mu - i\gamma)u_n + k_0v_n + k_1v_{n-1} = g|u_n|^2u_n, \\ (\mu + i\gamma)v_n + k_0u_n + k_1u_{n+1} = g|u_n|^2u_n, \end{cases} \quad (5)$$

bu yerda PT-simmetriya sharti quyidagi ko‘rinishda yoziladi:

$$u_n = \bar{v}_{-n}, \quad v_n = \bar{u}_{-n}. \quad (6)$$

(6)-tenglama PT-simmetriya shartining diskret ko‘rinishi. Juftiy ($n \rightarrow -n$) va vaqtga nisbatan simmetriya birgalikda qo‘llanilganda sistema o‘z-o‘ziga teng bo‘ladi.

2.2. Gauss profili bilan umumlashtirish

Endi biz PT-simmetrik DNS tenglamani (1) doimiy potensialni quyida berilgan Gauss potentsiali bilan almashtirish orqali umumlashtiramiz

$$i \frac{dq_n}{dt} + c_n(q_{n+1} - q_n) + c_{n+1}(q_{n-1} - q_n) - g|q_n|^2q_n + i(-1)^{n+1}V_nq_n = 0, \quad (7)$$

bu yerda

$$\begin{pmatrix} V_{2n} \\ V_{2n+1} \end{pmatrix} = \gamma_n, \quad \gamma_n = \gamma_0 \exp(-n^2\sigma^2). \quad (8)$$

σ potentsialning kengligi hisoblanadi. $\sigma \rightarrow 0$ bo‘lganda Konotop modeli hosil bo‘ladi, $\sigma \rightarrow \pm\infty$ da esa kuchayish-yutilish profili yo‘qoladi va sistema konservativ panjaraga soddalashadi.

Yana o‘zgaruvchilarni ajratish usuli (4) orqali quyidagi tenglamalar sistemasini olamiz:

$$\begin{cases} (\mu - i\gamma_n)u_n + k_0v_n + k_1v_{n-1} = g|u_n|^2u_n, \\ (\mu + i\gamma_n)v_n + k_0u_n + k_1u_{n+1} = g|v_n|^2v_n. \end{cases} \quad (9)$$

Chiziqli rejimda $g = 0$ bo‘lganda, (8)-tenglama quyidagi xos qiymatlar masalasiga aylanadi:

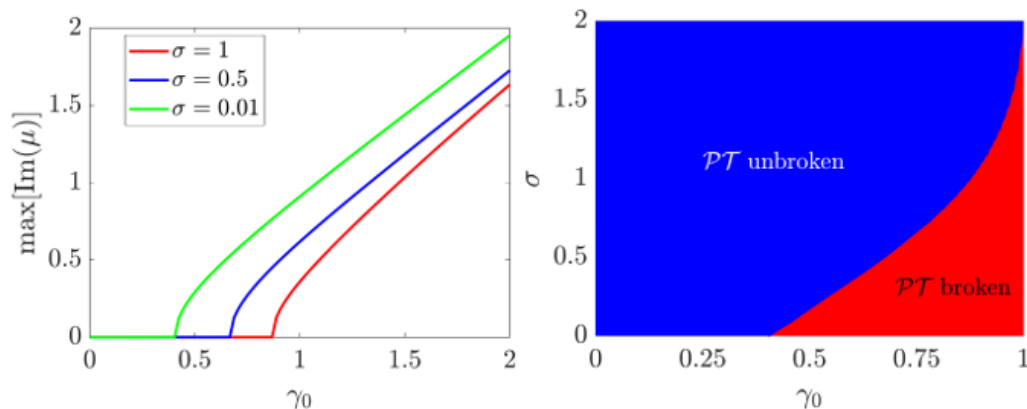


Figure 2. Dependence of the maximum value of the eigenvalue μ on γ_0 for different values of σ (left) and the regions in the γ_0 - σ plane where PT symmetry is preserved or broken (right).
Rasm 2. Turli σ qiymatlari uchun xos qiymat μ ning maksimal qiymati γ_0 ga bog'liqligi (chap) va γ_0 - σ tekislikdagi PT -simmetriya bajariladigan yoki bajarilmaydigan sohalar (ong).

$$\begin{cases} i\gamma_n u_n - k_0 v_n - k_1 v_{n-1} = \mu u_n, \\ -i\gamma_n v_n - k_0 u_n - k_1 u_{n+1} = \mu v_n. \end{cases} \quad (10)$$

Bu sistemani yechib, PT -simmetrik tizimning spektri (xos qiymatlari μ) topiladi (10)-tenglamadagi xos qiymatlar masalasini matritsa ko'rinishida quyidagicha ko'rinishda yozilishi mumkin:

$$H\omega = \mu\omega, \quad (11)$$

bunda H – cheksiz Hamiltonian, μ -xos qiymat, ω esa xos vektor. Butun panjara bo'ylab xos vektorlarni birlashtirgan ustun vektorni quyidagicha tavsiflash mumkin va H quyidagi ko'rinishdagi cheksiz matritsa-tridiagonal operator hisoblanadi:

$$H = \begin{cases} T_+, & n = m + 1, \\ D_n, & n = m, \\ T_-, & n = m - 1. \end{cases} \quad (12)$$

(12)-tenglamadagi 2×2 o'lchamdagi matritsalar esa quyida berilgan:

$$D_n = \begin{pmatrix} i\gamma_n & -k_0 \\ -k_0 & -i\gamma_n \end{pmatrix}, \quad T_+ = \begin{pmatrix} 0 & 0 \\ -k_1 & 0 \end{pmatrix}, \quad T_- = \begin{pmatrix} 0 & -k_1 \\ 0 & 0 \end{pmatrix}. \quad (13)$$

Bu matritsalar yuqori va pastki tridiagonal bog'lanishlarni ifodalaydi. Ekvivalent ravishda, xos qiymatlar masalasi har bir ω_n uchun yozilishi mumkin:

$$T_- \omega_{n-1} + D_n \omega_n + T_+ \omega_{n+1} = \mu \omega_n, \quad n \in \mathbb{Z}. \quad (14)$$

Biz (12)-tenglamadagi Hamiltonian H spektrini sonli usulda hisoblaymiz va xos qiymat μ ning mavhum qismi maksimal qiymatining γ_0 parametriga bog'liqligini chizamiz. (2-rasmda chap tomondagi panelga qarang). σ oshishi bilan kuchayish-susayish profili toraygani sayin, PT -faza o'tish chegarasi γ_0 ning kattaroq qiymatlariga siljiydi.

Bu xususiyat kutilgan, chunki kuchayish-susayish profilining lokalizatsiyasi oshishi bilan sistemada ermit bo'lmagan o'zaro ta'sirda qatnashayotgan panjara tugunlari soni kamayadi, natijada PT -simmetriyaning buzilishi uchun γ_0 ning kattaroq qiymatlari talab qilinadi.

1-rasm o'ng panelda esa $(\gamma_0 - \sigma)$ tekisligida PT -faza diagrammasini ko'rsatadi. Ko'k soha PT -simmetriya buzilmagan fazaga mos keladi, bu yerda xos qiymatlar to'liq haqiqiy bo'ladi; qizil soha esa kompleks xos qiymatlar paydo bo'lishi bilan aniqlanadigan PT -buzilgan fazani ifodalaydi.

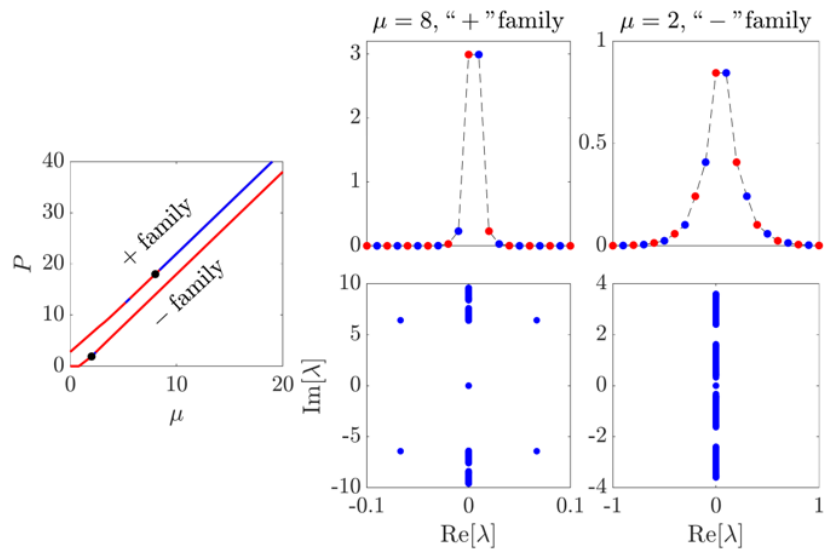


Figure 3. Separation of the soliton power $P(\mu)$ into linearly stable and unstable regions for the "+" and "-" soliton families (left), and stable and unstable soliton states for two different values of μ (upper right row) together with the corresponding eigenvalue λ spectra (lower right row).

Rasm 3. Soliton quvvati $P(\mu)$ ning solitonning "+" va "-" oilasi uchun chiziqli stabil va nostabil sohalariga ajralishi (chap) va ikkita turli μ qiymatlari uchun stabil va nostabil soliton holatlari (o'ng yuqori qator) va ularga mos bo'lgan xos qiymat λ spektri (o'ng pastki qator).

Faza chegarasi ushbu ikki rejimni ajratadi va PT-simmetriya buzilishi sodir bo'ladigan γ_0 va σ ning kritik qiymatlarini belgilaydi.

2.3. Diskret PT solitonlarning chiziqli stabiligi

Ushbu paragrafda [24] da berilgan usulga o'xshash tarzda diskret PT-simmetrik solitonlarning chiziqli barqarorligi o'rganiladi. Konotop modelidagi (5)-tenglamada γ_n yoziladi va markaziy dimerni $n = 0$ uyg'ongan, ya'ni $u_n, v_n \neq 0$ va qolgan dimerlar $n \neq 0$ uchun esa $u_n, v_n = 0$ deb hisoblaymiz. U holda yuqorida aytib o'tilgan tenglama quyidagi soddalashadi:

$$(\mu - i\gamma_0)u_0 + \bar{u}_0 = |u_0|^2 u_0, \tag{15}$$

bu yerda PT-simmetriya sharti $\bar{v}_0 = u_0$ dan foydalanildi. (14)-tenglamaning yechimini quyidagi ko'rinishda izlaymiz:

$$u_0 = A_{\pm} e^{i\phi_{\pm}}, \tag{16}$$

bu yerda $A_{\pm} = \mu \pm \sqrt{1 - \gamma_0^2}$, $\sin(2\phi_{\pm}) = -\gamma_0$ va $\cos(2\phi_{\pm}) = \pm \sqrt{1 - \gamma_0^2}$. Bundan keyin, ushbu ikki yechimni mos ravishda „++“ va „-“ oila deb nomlaymiz.

Ushbu diskret solitonning stabiligini tahlil qilish uchun maydon amplitudasiga quyidagi g'alayonlanish hadlari qo'shiladi

$$\begin{pmatrix} q_n(t) \\ q_{2n+1}(t) \end{pmatrix} = \left(\omega_n + e^{\lambda t} \psi_n + e^{\bar{\lambda} t} \bar{\varphi}_n \right) e^{-i(k_0+k_1+\mu)t} \tag{17}$$

(16) ifodada $\psi_n = (\psi_{u,n}, \psi_{v,n})^T$, $\varphi_n = (\varphi_{u,n}, \varphi_{v,n})^T$ va $|\psi_n|, |\varphi_n| \ll |\omega_n|$. Uni (7)-tenglamadagi DNS tenglamaga qo'yib, ψ_n va φ_n ni chiziqshatirib quyidagiga ega bo'lamiz.

Yuqori tartibli hadlarni tashlab yuborib, xos qiymatlar masalasini blokli ko'rinishda quyidagicha yoziladi:

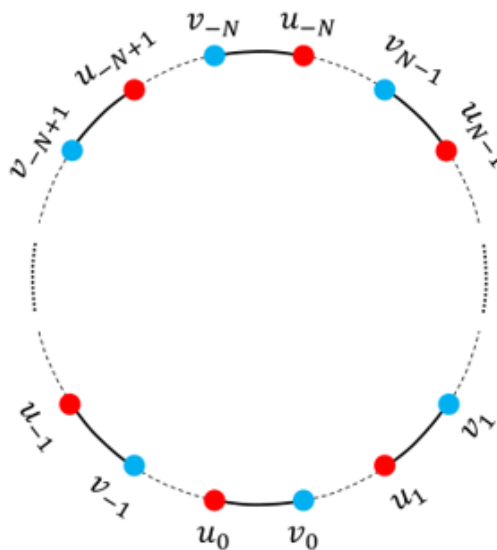


Figure 4. *PT*-symmetric ring
Rasm 4. *PT*-simmetrik halqa

$$\begin{pmatrix} T_- & 0 \\ 0 & T_- \end{pmatrix} \begin{pmatrix} \psi_{n-1} \\ \varphi_{n-1} \end{pmatrix} + \begin{pmatrix} L_n & G_n \\ -\bar{G}_n & -\bar{L}_n \end{pmatrix} \begin{pmatrix} \psi_n \\ \varphi_n \end{pmatrix} + \begin{pmatrix} T_+ & 0 \\ 0 & T_+ \end{pmatrix} \begin{pmatrix} \psi_{n+1} \\ \varphi_{n+1} \end{pmatrix} = i\lambda \begin{pmatrix} \psi_n \\ \varphi_n \end{pmatrix} \quad (18)$$

bu yerda $L_n = \text{diag}(2g|u_n|^2 - \mu, 2g|v_n|^2 - \mu) + D_n$, $G_n = g \text{diag}(u_n^2, v_n^2)$ va T_{\pm}, D_n lar (13) ifodada berilgan.

3-rasm chap panelda solitonning quvvatining

$$P(\mu) = \sum_{n=-\infty}^{\infty} (|u_n(\mu)|^2 + |v_n(\mu)|^2) \quad (19)$$

parametr μ ga bog'liqligi stabil va nostabil solitonlar uchun keltirilgan. Ushbu grafikdagi stabil va nostabil holda ikkita qora nuqta bilan berilgan μ ning qiymatlari uchun esa 3-rasm o'ng paneldagi 4 ta grafiklarda solitonlarning profillari va ularga mos bo'lsa xos qiymatlar masalasidagi (17) xos qiymat λ ning spektri ko'rsatilgan.

PT-simmetrik halqa

3.1. PT-simmetrik halqada chiziqli diskret Schrödinger tenglamasi

Ushbu paragrafda biz 4-rasmda tasvirlangan chiziqli rejimdagi ($g = 0$) *PT*-simmetrik halqani ko'rib chiqamiz. Bu yerda halqa $2N$ ta dimmerdan iborat bo'lib, ular $n \in \{-N, -N + 1, \dots, N - 1\}$ tarzda raqamlangan.

(10)-tenglama halqa bo'ylab yoziladi va chegaraviy dimerlar uchun bu tenglama $n = -N$ holatda quyidagicha qayta yoziladi:

$$\begin{cases} i\gamma_{-N}u_{-N} - k_0v_{-N} - k_1v_{N-1} = \mu u_{-N}, \\ -i\gamma_{-N}v_{-N} - k_0u_{-N} - k_1u_N = \mu v_{-N}, \end{cases} \quad (20)$$

va $n = N - 1$ uchun quyidagicha:

$$\begin{cases} i\gamma_{N-1}u_{N-1} - k_0v_{N-1} - k_1v_N = \mu u_{N-1}, \\ -i\gamma_{N-1}v_{N-1} - k_0u_{N-1} - k_1u_{-N} = \mu v_{N-1}. \end{cases} \quad (21)$$

5-rasmda *PT*-simmetrik halqadagi xos dispersion munosabat keltirilgan. Halqaning chekli o'lchami tufayli energiya spektri diskretlashadi va *PT*-buzilish nuqtasi uchun kritik γ_0 qiymati o'rni halqa o'lchamiga bog'liq bo'lib qoladi. Rasmda, turli σ lar uchun xos qiymatlarning μ haqiqiy

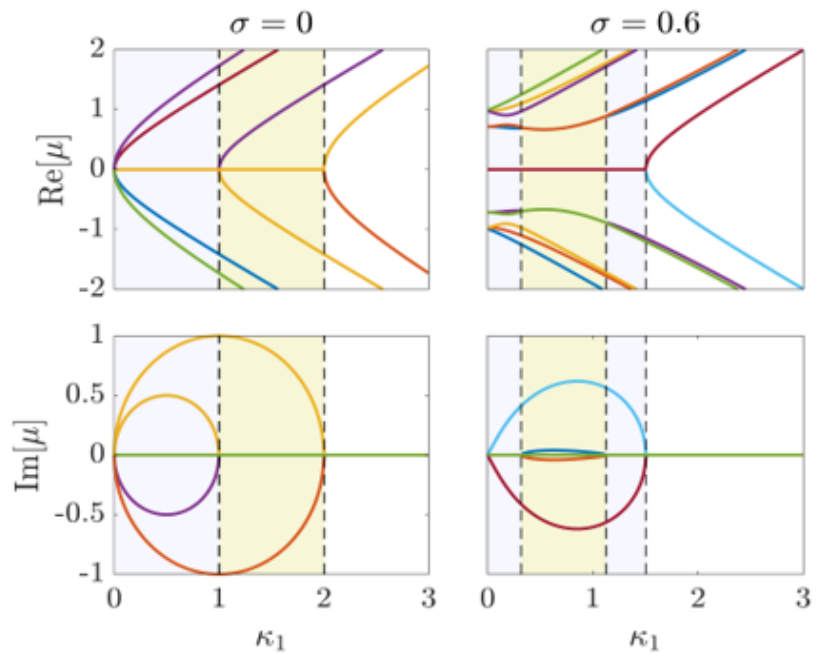


Figure 5. The eigenvalue spectrum or PT-phase diagram in a PT-symmetric ring is presented
Rasm 5. PT-simmetrik halqadagi xos qiymatlar spektri yoki PT-faza diagrammasi keltirilgan

va mavhum qismlarining γ_0 ga nisbatan o'zgarishi, shuningdek PT-simmetriyaning buzilmagan (haqiqiy spektr) va buzilgan (mavhum qisimli spektr) sohalari chegaralari aks ettirilgan.

6-rasmda PT-simmetrik halqa ko'rinishidagi panjara modelida nochiziqli hol $g \neq 0$ hol uchun soliton profillari tasvirlangan. Bunda "+" oila chiziqli nostabil va "-" oila uchun chiziqli stabil soliton yechimlari keltirilgan. Pastdagi qatordagi ikki grafiklarda esa shu solitonlarga mos bo'lgan chiziqli stabillik spektri ko'rsatilgan.

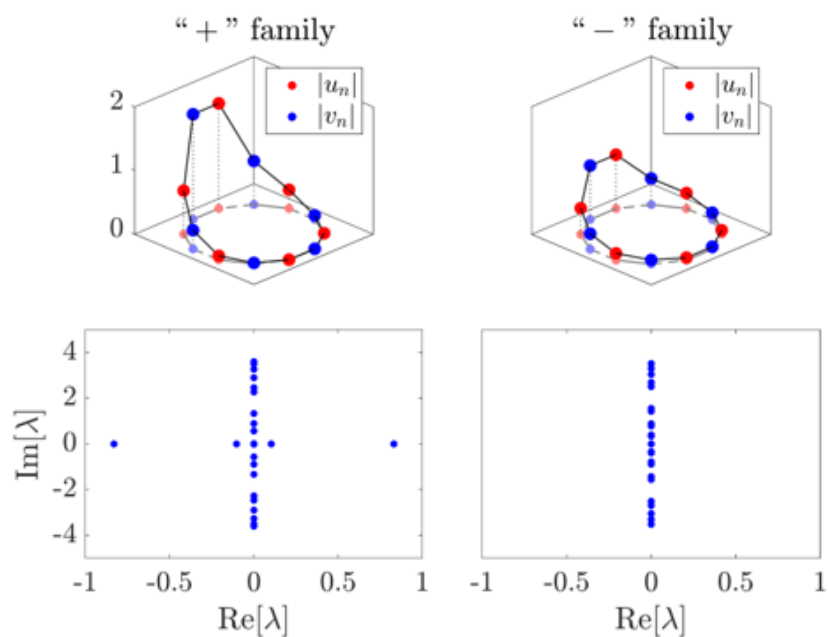


Figure 6. The soliton states belonging to the "+" and "-" families in a PT-symmetric ring and their corresponding linear stability spectra

Rasm 6. PT-simmetrik halqada "+" va "-" oilaga tegishli solitonning holati va ularga mos bo'lgan chiziqli stabillik spektri

Xulosa

Ushbu tadqiqotda PT-simmetrik diskret nochizikli Schrödinger tenglamasi asosida halqa bir o'ldamli panjara va halqa shaklidagi panjara uchun Gauss ko'rinishidagi mavhum potentsial ta'sirida chiziqli holda dispersion munosabatlar va nochiziqli holda esa solitonlarning hosil bo'lishi, ular uchun chiziqli stabillik tahlillari o'rganildi. Quyidagi asosiy ilmiy natijalarga erishildi:

Gauss profilidagi kuchayish-susayish $\gamma_n = \gamma_0 \exp(-n^2 \sigma^2)$ orqali umumlashtirilgan modelda PT-simmetriyaning buzilmagan fazasi (energiya haqiqiy) va buzilgan fazasi (energiya kompleks) holatlari o'rtasidagi faza chegarasi (γ_0, σ) tekisligida sonli usulda qurildi. Aniqlan, σ ortishi (kuchayish-susayish profilining torayishi) bilan PT-buzilmagan faza hududi kichik γ_0 qiymatlari tomon siljiydi. (17) tenglama orqali ifodalangan chiziqli stabillik tahlili asosida PT-simmetrik solitonlarning stabillik xususiyati o'rganildi.

PT-simmetrik halqada chegaraviy shartlar tufayli xos qiymatlar spektri diskretlashdi va PT-buzilish nuqtasining o'rni halqaning o'ldamiga bog'liqligi ko'rsatildi. Halqa modeli cheksiz panjaradan farqli ravishda PT-faza o'tishida qo'shimcha kvantlanish effektlarini keltirib chiqaradi.

Olingan natijalar PT-simmetrik optik to'ldin o'tkazgich massivlarida signallarni bir kanaldan ikkinchisiga uzatish, optik izolyatorlar va bir tomonlama o'tkazgichlarni loyihalashda qo'llanilishi mumkin.

Mualliflarning hissalari

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Authors' contribution

Author Contributions: Conceptualization, M.A.; methodology, Sh.T. and A.F.; software, A.F.; validation, Sh.T., A.F. and M.A.; formal analysis, Sh.T. and A.F.; investigation, Sh.T. and A.F.; resources, M.A.; data curation, A.F.; writing — original draft preparation, Sh.T.; writing — review and editing, M.A.; visualization, Sh.T. and A.F.; supervision, M.A.; project administration, M.A.; funding acquisition, M.A. All authors have read and agreed to the published version of the manuscript.

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Ethics approval

Not applicable. This study did not involve humans or animals; therefore, ethical approval was not required.

Ma'lumotlar mavjudligi to'g'risidagi bayonot

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Data Availability Statement

No new experimental data were generated in this study. All essential data supporting the findings of this research are included within the article and its figures.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Qisqartmalar

PT	Parity–time symmetry
DNLS	Discrete nonlinear Schrödinger equation
NLS	Nonlinear Schrödinger equation
ODE	Ordinary differential equation
PDE	Partial differential equation
1D	One-dimensional
ML	Machine learning emas, bu maqolada ishlatilmagan bo'lsa kiritilmaydi
μ	Eigenvalue parameter
λ	Linear stability eigenvalue
γ_0	Gain–loss strength
σ	Width of the Gaussian gain–loss profile
$P(\mu)$	Soliton power

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Nashriyot javobgar emas/ eslatmasi:

Barcha nashrlarda keltirilgan bayonotlar, fikrlar va ma'lumotlar faqat mualliflar va ishtirokchilarga tegishlidir, na Jurnal va na muharrirlar. Jurnal va muharrirlar, mazkur kontentda keltirilgan har qanday g'oyalari, usullari, ko'rsatmalar yoki mahsulotlar natijasida insonlar yoki mulkka yetkazilgan har qanday zarar uchun javobgar emas.

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